A STUDY OF NUCLEAR PION ABSORPTION

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ABSTRACT

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Various aspects of pion-nucleus interactions are studied. The S-wave pion optical potential is studied up to second order investigating effects due to the modification of the pion propagator. That is, the ordinary off-shell pion propagator is replaced by one which includes the effects of the average field generated by other nucleons.

S and P wave pion absorption in a finite nucleus and in nuclear matter are studied. Emphasis is on a rescattering absorption mechanism which turns out to be equivalent to the quasi-free deuteron model. The results are improved by including medium effects in the pion propagator, as in the S-wave scattering case.

The possibility of nuclear opalesence and pion condensation phenomena are discussed. The isospin dependence of the P-wave absorption in He isotopes in the resonance region is explained by the rescattering process. Finally the question of the region in which absorption takes place and the problem of overlapping sources is discussed.

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Chapter I

Introduction

Information about nuclear structure is usually obtained from nuclear reactions. The pion has become an important probe, with the production of high quality meson beams. Because it has zero spin, its interaction is simpler than that of spin 1/2 or spin 1 probes.

The pion has a small mass, m_{π} = 140 Mev = 0.15 m_{N} , so it is relativistic even at low energies. It can also be absorbed by a nucleus. In these two respects it is similar to a photon. However the pion strongly interacts with nuclear matter, it also exhibits strong resonance behavior due to the formation of the Δ resonance in the P wave pion-nucleon channel.

In this thesis the emphasis is on a particular mechanism for pion absorption, the rescattering mechanism where a pion is absorbed by a pair of nucleons, being scattered by one and absorbed by the other.

There are two such mechanisms, the first being S wave pion scattering followed by absorption, and the second involving P wave pion scattering. The first is predominant at threshold, the second, which goes via the creation of a virtual Δ resonance, is the most important for pion kinetic energies > 50 Mev, including the resonance region. Here both π and ρ mesons are included in intermediate states.

Experiments on pion absorption by the He isotopes in the resonance region, show that the ratio of absorption on $T = 0$ nucleon pairs to $T = 1$ nucleon pairs is very large, and this is explained in a natural way by P wave rescattering in the present model, thus justifying the old quasi-deuteron absorption model.

However absorption parameters in nuclear matter calculated by fitting the parameters of the model to pion absorption by free deuterons generally give results which are too small as compared with experiment. One difference between absorption by a free deuteron and two nucleons in nuclear matter is that in nuclear matter,the pion propagator is affected by the average field of the other nucleons, expressible in terms of the pion self-energy in nuclear matter. This is not a small effect, and occurs also in other physical processes. It could give rise to phenomena like pion condensation or nuclear opalescence. These processes do not appear to happen at normal nuclear densities. The strong momentum dependent P wave pion interaction with nucleons which would tend to give rise to such phenomena is damped by processes at short internucleon separation, usually parameterized by the Landau parameter g', or equivalently by the Lorenz-Lorentz-Ericson-Ericson parameter, λ ,

(LLEE) , [Ref. I.1] which derives its name from an analogous process in the propagation of an electromagnetic wave in a polarizable medium. Non observation of nuclear opalescence in other experiments means the $g' > 0.5$ or $\lambda > 1.5$.

The parameters of the theory are the coupling constants at the interaction vertices, the form factors Λ_π , Λ_ρ at the vertices, which give the effective size of the nucleons or isobars acting as sources for the meson fields, and the LLEE parameter λ . This is all part of the standard $\pi + \rho$ phenomenology involved in other areas of nuclear physics such as the excitation of spin-isospin flip states in inelastic proton scattering.

The use of finite size sources, where the size is an effective one, and may be due to complicated processes at short distance, indicates that the calculation is likely to be invalid if the sources overlap, so the region of nucleon separation in which absorption occurs is investigated. The relation, if any, between source effective size and the bag model of hadrons is briefly discussed. Clearly an understanding of the parameters λ or g' , would involve understanding the properties of overlapping bags. A recent attempt in this direction has been made by Weise [Ref. I.2].

Two nucleon absorption may not be the whole story. Recent experiments near resonance indicate that cluster absorption involving the excitation of two Δ 's may be important at resonance energies for large nuclei. This will not be considered here. In any case it involves the same basic processes, which have been used to estimate the ratio of 4-body to 2-body terms.

This dissertation is organized as follows. In Chapter II the S wave part of the optical potential for pions is discussed. Calculations are carried out up to second order, including nuclear medium effects in the second order term, but excluding absorption. The purpose is to investigate the origin of the strong repulsive nature of this part of the potential observed empirically.

In Chapter III pion absorption by S wave rescattering is investigated for both nuclear matter and ^{4}He . The nuclear medium effect is calculated, as are the isotopic ratios and angular distribution of emitted nucleons. Some of this has already been published by the present investigator.

In Chapter IV P wave rescattering is investigated for both nuclear matter and ^{4}He . Nuclear medium effects are calculated, as is the isotopic ratio for emitted nucleons. The latter has also been published by the present author.

In Chapter V the question of the region in which absorption takes place and the problem of overlapping sources is briefly discussed. A calculation of the absorption parameter in an alternative form of the P wave optical potential (Δ -h model) is also presented.

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Chapter II

πN SCATTERING

The kinematical region of interest is pion kinetic energies of $\lesssim$ 220 Mev . In this region, the πN scattering amplitude consists of at most two partial waves, L = 0 and 1. P wave (L = 1) scattering, dominates due to the formation of the intermediate resonance state, Δ , in spin and isospin 3/2 channels. The S wave (L= 0) channel, although small, is important at low energies. In this section S wave πN scattering is studied.

2-1 S Wave πN Scattering

S wave pions in principle could be scattered by the meson (pion) cloud surrounding a nucleon and also by the " bare " (core) nucleon,

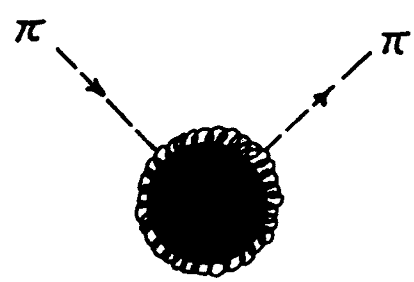

Figure II.1 Scattering of a pion by a nucleon

or equivalently

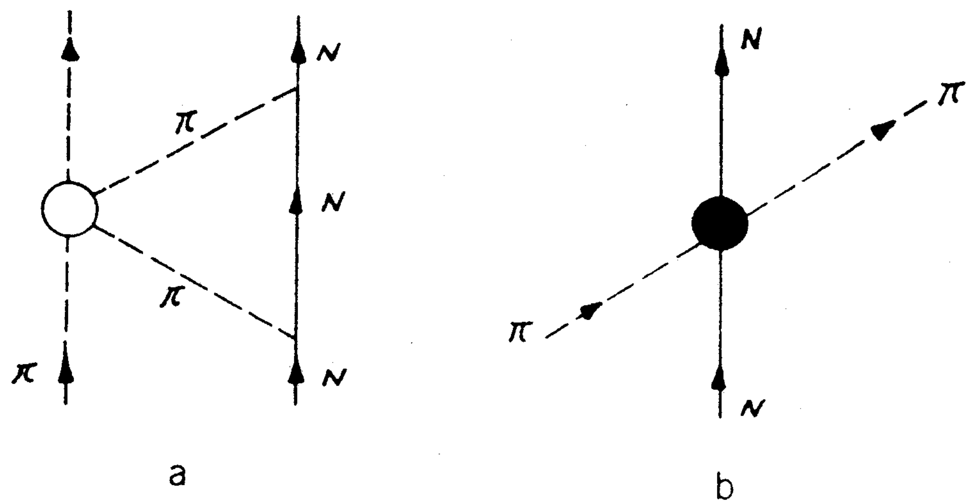

Figure II.2a Scattering of a pion by meson cloud
Figure II.2b Scattering of a pion by nucleon core

Figure II.2a shows the scattering of the incident pion by the pion cloud and Figure II.2b shows the scattering by the core.

The contribution of Figure II.2a is small and may be ignorerd [Ref. II.1] . But part 2b itself, is composed of at least three dynamical channels.

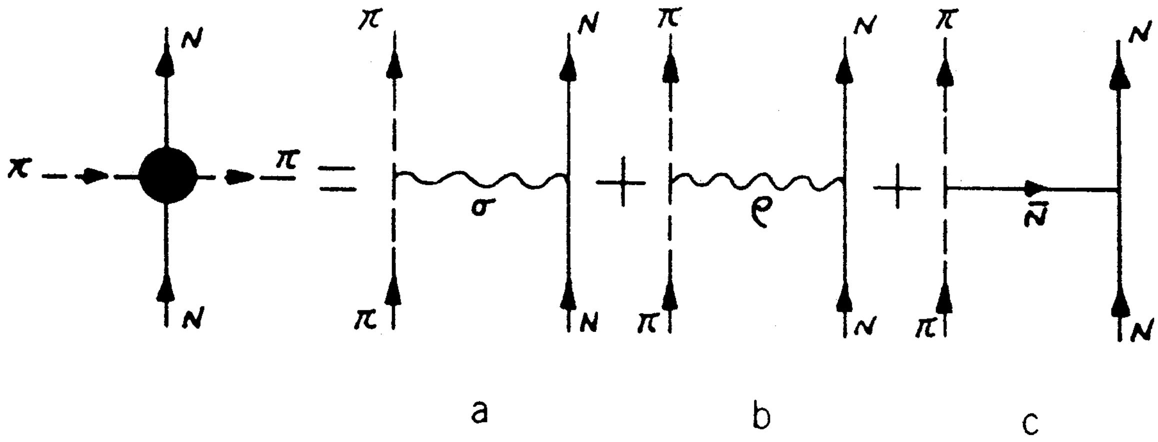

Figure II.3 Three dynamical channels of pion-core interaction

The channel in Figure II.3a, due to the exchange of the scalar σ meson, contributes to the isoscalar part of πN scattering. Diagrams 3b and 3c are due to vector meson ρ and anti-nucleon $\bar{N}$ exchange and describe the iso-vector part.

For low energy scattering it is sufficient to use a phenomenological interaction [Ref. II.2]

$$H_{\pi\pi NN} = \frac{4\pi}{\mu}\lambda_1\bar{\psi}\,\vec{\phi}\cdot\vec{\phi}\,\psi + \frac{4\pi}{\mu^2}\lambda_2\bar{\psi}\,\vec{\tau}\cdot(\vec{\pi}\times\vec{\phi})\psi \qquad (2\text{-}1\text{-}1)$$

where ψ is the nucleon field operator and $\vec{\phi}$ and $\vec{\pi}$ ($=\partial_t\vec{\phi}$) are the pion field and its conjugate operator respectively, and $\vec{\tau}$ is the nucleon isospin operator. This interaction is parameterized through two scattering lengths, λ_1 and λ_2 which are given [Ref. II.3]

$$\lambda_1 = 0.003 \pm 0.001$$
$$\lambda_2 = 0.05 \pm 0.001 \qquad (2\text{-}1\text{-}2)$$

By using these two scattering lengths the S wave πN, S_{31} phase shift in isospin 3/2 and spin 1/2 channel up to 50 Mev and S_{11} phase shift in isospin 1/2 and spin 1/2 channel up to 250 Mev pion's lab kinetic energy are well reproduced.

To improve the S_{31} phase shifts for higher energies, energy dependent scattering lengths are proposed [Ref. II.4]

$$\lambda_1 = 0.003 + 0.0344\frac{T}{\mu} - 0.0058\left(\frac{T}{\mu}\right)^2$$

$$\lambda_2 = 0.05 - \frac{\mu}{\omega}\left[0.0334\frac{T}{\mu} - 0.058\left(\frac{T}{\mu}\right)^2\right] \qquad (2\text{-}1\text{-}3)$$

where T is the pion's kinetic energy in the laboratory system.

2-2 π A First Order S Wave Optical Potential

Adopting the multiple scattering approach [Ref. II.5] to study the πA scattering problem, the basic process would be the scattering of an incident pion via single bound nucleons. To lowest order [Ref. II.6], the optical potential U_{opt} , is parameterized in the following form

$$2\omega U_{opt} = -4\pi b_o^{(1)} \rho \qquad (2\text{-}2\text{-}1)$$

where $b_o^{(1)}$ is the first order S wave scattering parameter and ρ is the nuclear density.

From the Hamiltonian Equation (2-1-1), the πN scattering amplitude $t^{\pi N}$, in momentum space for this process is

$$t^{\pi N} = \frac{4\pi}{\mu}\left[2\lambda_1 - i(\omega+\omega') \frac{\lambda_2}{\mu} \vec{t}.\vec{\tau} \right] \qquad (2\text{-}2\text{-}2)$$

where ω' is the rescattering pion's energy and $\vec{t}$ is the pion's isospin operator.

The Fourier transform of the expectation value of this operator between initial and final nuclear states expressed in momentum space in the elastic scattering channel i.e. $\omega = \omega'$ results in the spatial representation of the optical potential

$$2\omega U_{opt} = \frac{8\pi}{\mu}\left[\lambda_1 \rho - i\frac{\omega}{\mu} \lambda_2 t_3 (\rho_p - \rho_n) \right] \qquad (2\text{-}2\text{-}3)$$

In nuclear matter and / or symmetric nuclei, where the proton and neutron densities are identical, Equation (2-2-3) would reduce to the more compact form

$$2\omega U_{opt} = \frac{8\pi}{\mu}\lambda_1 \rho \qquad (2\text{-}2\text{-}4)$$

By comparing this equation with Equation (2-2-1), the S wave scattering parameter becomes, in terms of the fitted scattering length λ_1 ,

$$b_0^{(1)} = -\frac{2}{\mu}\lambda_1 \qquad (2\text{-}2\text{-}5)$$

From this result one may conclude that the first order S wave scattering parameter $b_0^{(1)}$ in the kinematical region of interest is independent of incident pion energy, and since it is negative, it leads to a repulsive optical potential. But since λ_1 is very small, the first order S wave potential would not have any significant effect. Hence, higher order S wave effects should be considered.

2-3 πA Second Order S Wave Optical Potential

In the second order S wave πA multiple scattering process, the incident pion gets the chance of being scattered twice by two different nucleons. The incident on shell pion gets scattered by one of the nucleons, propagates off shell through the nucleus and is rescattered by a second nucleon.This process is shown diagramatically in Figure II.4

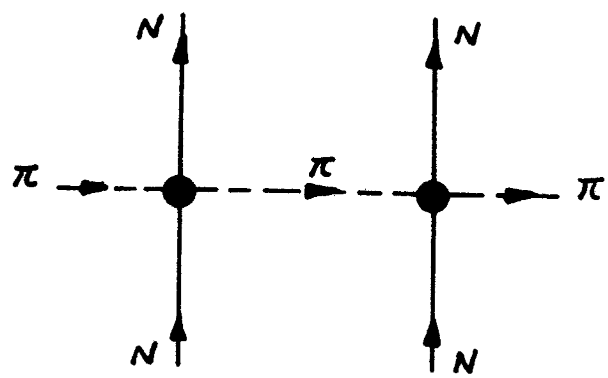

Figure II.4 Second order S wave pion scattering

Applying the interactive Hamiltonian Equation (2-1-1) twice in this second order diagram in the elastic channel, the scattering amplitude becomes

$$T^{(2)} = (4\pi)^2 G(\omega, \vec{k}'') \frac{2}{\mu^2} \left[\lambda_1^2 + \frac{2}{3} \left(\frac{\omega}{\mu}\right)^2 \lambda_2^2 \, \vec{\tau}^1 \cdot \vec{\tau}^2 \right] \tag{2-3-1}$$

where $G(\omega, \vec{k}'') = \dfrac{-i}{\mu^2 + k''^2 - \omega^2}$ is the off-shell propagator, $\vec{k}''$ is the pion momentum, and $\vec{\tau}^1$ and $\vec{\tau}^2$ are first and second nucleon's isospin operators.

Fourier transformation of this amplitude gives the optical potential operator

$$2\omega U_{opt} = \int T^{(2)} e^{i\vec{k}''\cdot\vec{r}} \frac{d\vec{k}''}{(2\pi)^3} \tag{2-3-2}$$

where the $\vec{r} = \vec{r}_1 - \vec{r}_2$ is the relative separation vector of the two nucleons. The nuclear matrix element of this operator involves the exchange part of the two body density operator

$$\rho_{ex} = \frac{1}{A(A-1)} \frac{9}{16} P_{12}^{\sigma} P_{12}^{\tau} \left[\frac{J_1(K_F|\vec{r}_1 - \vec{r}_2|)}{K_F|\vec{r}_1 - \vec{r}_2|} \right]^2 \rho(r_1)\rho(r_2) \quad (2\text{-}3\text{-}3)$$

where A is the atomic number of the nucleus and P_{12}^{σ} and P_{12}^{τ} are the spin and isospin exchange operators

$$P_{12}^{\sigma} = \frac{1}{2} (1 + \vec{\sigma}^1 \cdot \vec{\sigma}^2)$$

$$P_{12}^{\tau} = \frac{1}{2} (1 + \vec{\tau}^1 \cdot \vec{\tau}^2) \quad (2\text{-}3\text{-}4)$$

$K_F = 1.4\ fm^{-1}$ is the Fermi momentum, and ρ is the nuclear density. For nuclear matter i.e. $\rho = 2/3\pi^2 K_F^3$ and for threshold pions the following second order S wave optical potential parameter results

$$b_0^{(2)} = - \frac{6}{\pi} \frac{K_F}{\mu^2} (\lambda_1^2 + 2\lambda_2^2) \quad (2\text{-}3\text{-}5)$$

Above threshold, analytical calculation is not possilble and integration has be carried out numerically, [Ref. II.7] .

One may compare this result with the first order parameter Equation (2-2-5). The ratio of $b_0^{(2)} / b_0^{(1)} \cong 3$

shows the significance of second order effects. Identical signs of $b_0^{(2)}$ and $b_0^{(1)}$ stress the coherent nature of these two effects in generating a stronger repulsive optical potential, which is needed for better fits to S wave pionic atom level widths.

2-4 Modification of πA Second Order S Wave Optical Potential

Even restricting the multiple scattering to second order, the method of the previous section is an approximation to what actually happens. Although only two nucleons get involved actively in the second order rescattering process and the rest of the nucleus acts as a spectator, it does have an effect. That is the off shell rescattered pion does not propagate between two isolated nucleons in vacuum, but in a medium which exerts an average field on the pion.

Usually this effect is refered to as the pion's " Self Energy " Π , [Ref. II.8] . It should be added explicitly to the pion propagator

$$G(\omega, \vec{k}'') = \frac{-i}{\mu^2 + k''^2 - \omega^2} \rightarrow \frac{-i}{\mu^2 + k''^2 - \omega^2 + \Pi} \qquad (2\text{-}4\text{-}1)$$

Diagramatically the self energy contribution is depicted in Figure II.5

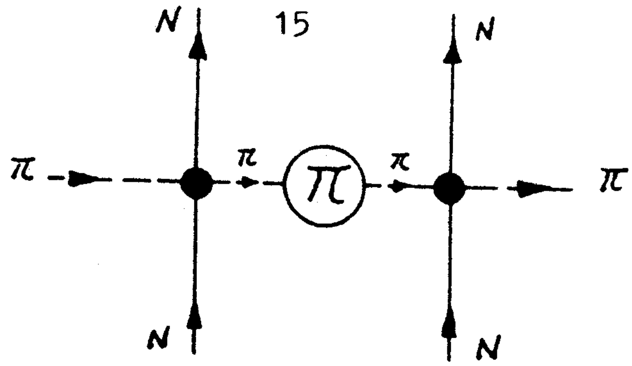

Figure II.5 The second order S wave pion rescattering mechanism with inclusion of medium effects

In brief, [Ref. II.8], the off shell propagation of the pion in nuclear matter is considered as the excitation and de-excitation of an infinite number of virtual isobar-hole states (Δ - h) . The complete pion self energy is obtained by summing the chain of Δ-h diagrams connected by the reduced isobar-hole interactions. Analytically it is given by

$$\Pi(k) = \Pi_0(k) \Big/ 1 - \frac{\lambda}{3} \frac{\Pi_0(k)}{k^2 f_\pi^2(k)} \qquad (2\text{-}4\text{-}2)$$

Π_0 is the self energy of pion due to a single Δ-h excitation

$$\Pi_0(k) = -\frac{4}{9}\left(\frac{f_{\pi N \Delta}}{\mu}\right)^2 k^2 f_\pi^2(k)\, \rho \left(\frac{1}{D_1} + \frac{1}{D_2}\right) \qquad (2\text{-}4\text{-}3)$$

with the energy denominators

$$D_1 = m_\Delta - m_N - \omega \qquad (2\text{-}4\text{-}4)$$

$$D_2 = m_\Delta - m_N + \omega$$

m_Δ and m_N are the Δ and nucleon masses respectively, the $\pi N \Delta$ coupling constant $f^2_{\pi N \Delta}/4\pi = 0.32$ [Ref. II.9] and K and ω are the incident pion's momentum and energy, ρ is the nuclear density and $f_\pi(k)$ is the monopole form factor,

$$f_\pi(k) = \Lambda^2 - \mu^2 / \Lambda^2 + k^2 - \omega^2 \qquad (2\text{-}4\text{-}5)$$

Λ is the cut off mass parameter [Ref. II.3] and λ is the LLEE parameter [Ref. II.10]: its range should be confined to $1 < \lambda < 2$, [Ref. II.11] . This takes account of, among other things, the short range correlation between nucleons.

$\Pi(k)$ can be written as

$$\Pi(k) = -k^2 P f_\pi^2(k) \qquad (2\text{-}4\text{-}6)$$

Inclusion of this in the propagator means

$K''^2 \rightarrow k''^2(1-P f_\pi^2)$ and, as at threshold $\omega^2 \simeq \mu^2$, $f_\pi = 1.$,

this means approximately that $1/k''^2 \longrightarrow 1/k''^2(1-P)$ i.e.

the propagator is amplified by a factor $1/1-P$.

Putting in the numerical values, at threshold,

$P \simeq 1.15/1+0.38\lambda$. Note that if λ was small, $1/1-P$

could be very large.

Performing the calculation, the second order S wave scattering parameter then becomes [Ref. II.12]

$$b_0^{(2)} = -\frac{6}{\pi}\frac{K_F}{\mu}(\lambda_1^2+2\lambda_2^2)\frac{1}{1-P}\left[1-\frac{8\pi P K_F^2}{54(1-P)(\Lambda^2-\mu^2)}\right] \quad (2\text{-}4\text{-}7)$$

Since P carries all the information about the nuclear medium, the overall many body contribution to the problem, i.e. the $1/1-P$ factor, amplifies the strength of the scattering parameter $b_0^{(2)}$. The second term in Equation (2-4-7) simply shows the effect of the form factor.

By extending the above formalism beyond pion threshold, we have shown that the second order scattering parameters become

$$Re\, b_0^{(2)} = -\frac{6}{\pi^2}\frac{K_F}{\mu}(\lambda_1^2+2\lambda_2^2)\frac{1}{1-P}\left[k_F^2\, Re\, I - \frac{8\pi P K_F^2}{9(1-P)(\Lambda^2-\mu^2)}\right] \quad (2\text{-}4\text{-}8a)$$

$$Im\, b_0^{(2)} = -\frac{6}{\pi^2}\frac{K_F}{\mu}(\lambda_1^2+2\lambda_2^2)\frac{1}{1-P}K_F^2\, Im\, I \quad (2\text{-}4\text{-}8b)$$

where

$$K_F^2 \, Re\, I = 4\pi \int_0^\infty \frac{1}{x} J_1^2(K_F x) \frac{\sin(2Kx)}{2Kx} dx \qquad (2\text{-}4\text{-}9a)$$

$$K_F^2 \, Im\, I = 4\pi K \int_0^\infty J_1^2(k_F x) \left(\frac{\sin(Kx)}{Kx}\right)^2 dx \qquad (2\text{-}4\text{-}9b)$$

By choosing Λ_π = 1.2 Gev and λ = 1.6 , which is the appropriate value to use for pion absorption in nuclear matter [Ref. II.13], discussed in detail in Chapter III, the combined first and second order S wave scattering parameter at threshold becomes $-0.066\mu^{-1}$ which is about 2.5 times stronger than the original value given by $b_0^{(1)} + b_0^{(2)}$ without medium effect . This extra repulsive strength in the optical potential gives better fits to level shifts of pionc atoms.

At this stage we would like to justify the validity of the sharp resonance approximation which was used to construct the pion self-energy. That is, the direct channel's energy denominator D_1 , given by Equation (2-4-5) , does not include the isobar's width. The exact D_1 should read

$$D_i = m_\Delta - m_N - \frac{1}{2}\mu - \frac{i}{2}\Gamma_\Delta \qquad (2\text{-}4\text{-}10)$$

where

$$\Gamma_\Delta = \frac{2}{3}\left(\frac{f_{\pi N\Delta}}{4\pi}\right)^2 \frac{1}{\mu^2} k^3 \qquad (2\text{-}4\text{-}11)$$

and K is the pion's momentum [Ref. II.14] .

At low energies, say $T_{\pi} \cong 50$ Mev , the maximum value of $\Gamma_{\Delta}/2 = 0.02\mu$, which by comparing to the real part of D_1 can be neglected. At higher energies the isobar's width becomes larger. Consequently, for more accurate calculations, one should take it into account.

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Chapter III

PION ABSORPTION

Introduction

As pointed out in Chapter I, the distinct feature of a pion which distinguishes it from other probes, is its absorption by a nucleus. A pion cannot be absorbed by a free nucleon. It can be absorbed by a bound nucleon, but more likely by a cluster of nucleons, which contains at least two nucleons.

Experimental data [Ref. III.1] in the kinematical region of interest indicates that pion absorption in a typical light nucleus often results in two-nucleon emission. So a two-body microscopic absorption model is investigated.

The πA scattering amplitude in the pion's energy region that we are interested in containes only two partial waves i.e. $L = 0,1$. So, the two-nucleon absorption process is studied separately for S and P wave pion rescattering. In the next section details of the S wave absorption are presented.

3-1 S Wave Pion Absorption in Nuclear Matter

True absorption by S wave pion rescattering is parametrized by an optical potential term quadratic in the nuclear density [Ref. III.2] , which indicates absorption via a two-body mechanism, so

$$2\omega U_{opt} = -4\pi B_0 \rho^2 \tag{3-1-1}$$

Here ω is the incident pion's energy, ρ is the nuclear density and B_0 is the absorption parameter. On the other hand from the multiple scattering pion-nucleus series expansion the optical potential may be written as

$$\langle K|U|k'\rangle = \langle k'|V|k\rangle + \langle k'|V \frac{1}{E - H_0 + i\epsilon} V|k\rangle \tag{3-1-2}$$

where the optical potential $\langle k'|U|k\rangle$ is usually denoted by U_{opt}, and $\langle k'|V|k\rangle$ is the nuclear matrix element of the elementary single pion-nucleon interaction operator. The pion momenta are K and K' .

By inserting a complete set of nuclear states $\sum_f |f\rangle\langle f| = 1$ in the second order optical potential and splitting the nuclear Green's function into its dispersive and absorptive parts it becomes

$$U^{(2)}_{opt} = \frac{1}{\Omega} \sum_{f\neq i} T_{if} \left[\frac{P}{E_i + \omega - E_f} - i\pi\,\delta(E_i + \omega - E_f) \right] T_{fi} \tag{3-1-3}$$

Here T_{fi} is the matrix element of the two-nucleon pion absorptive operator between the nuclear states i and f of energy E_i and E_f respectivly and Ω is the nuclear volume. For a Hermitian absorption operator Equation (3-1-3) beomes

$$U^{(2)}_{opt} = \frac{1}{\Omega} \sum_{f \neq i} \left[\frac{1}{E_i + \omega - E_f} - i\pi\,\delta(E_i + \omega - E_f) \right] |T_{fi}|^2 \quad (3\text{-}1\text{-}4)$$

By comparing Equations (3-1-1) and (3-1-4) we get

$$Re\,B_o = -\frac{\omega}{2\pi\rho^2\Omega} \sum_{f \neq i} |T_{fi}|^2 \frac{P}{E_i + \omega - E_f}$$

$$Im\,B_o = \frac{\omega}{2\rho^2\Omega} \sum_{f \neq i} |T_{fi}|^2\, \delta(E_i + \omega - E_f) \quad (3\text{-}1\text{-}5)$$

In most of the S wave calculations [Ref. III.3-4] the absorption operator is based on the diagram given below

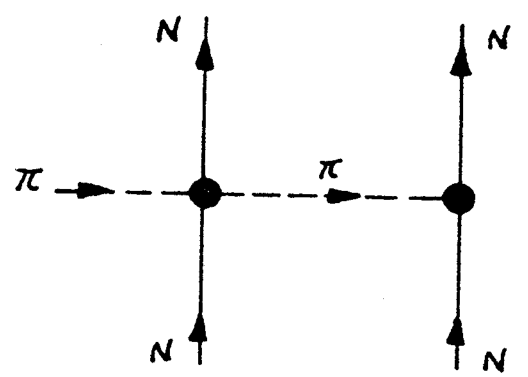

Figure III.1 S wave pion absorption via two-body mechanism

Here the S wave pion is scattered by one of the nucleons and absorbed by the second one. The scattering vertex can be described by Equation (2-1-1) and the absorption vertex by the usual $H_{\pi NN}$ Hamiltonian density [Ref. III.5]

$$H_{\pi NN} = f_{\pi NN}/\mu \ \bar{\Psi} \gamma_5 \vec{\Phi} \Psi . \vec{\tau} \tag{3-1-6}$$

By applying the standard Feynman rules, the absorption operator becomes

$$T = -i \frac{8\pi}{\mu^2} f_{\pi NN} \frac{1}{\sqrt{\omega}} \frac{\vec{\sigma}^2 \cdot \vec{k}'}{\mu^2 + k'^2 - \omega'^2} \left[\lambda_1 \tau_+^2 - i \frac{\lambda_2}{2\mu} (\omega + \omega') (\vec{\tau}' \times \vec{\tau}^2)_+ \right] \tag{3-1-7}$$

The calculation of the matrix element of T is straight forward. Two cases are considered. The first is nuclear matter, where the Fermi gas model is used to construct the initial pair wave function [Ref. III.3]. The second deals with finite nuclei, where harmonic oscillator wavefunctions are used to construct the initial pair wave function [Ref. III.4] . In both cases, the final pair nucleon wave functions are described in terms of out going plane waves, that is, distortion is not included.

For threshold pions, nuclear matter calculation leads to Re $B_0 = 0.008\mu^{-4}$ and Im $B_0 = 0.02\mu^{-4}$ [Ref. III.3,6] . The energy dependence of the latter is shown in Figure III.2 , and the energy dependence of Re B is given in [Ref. III.3]. As compared to the value obtained from pionic atom level widths, Im $B_0 = 0.042\mu^{-4}$ [Ref. III.7], the absorption value is too small by a factor of two.

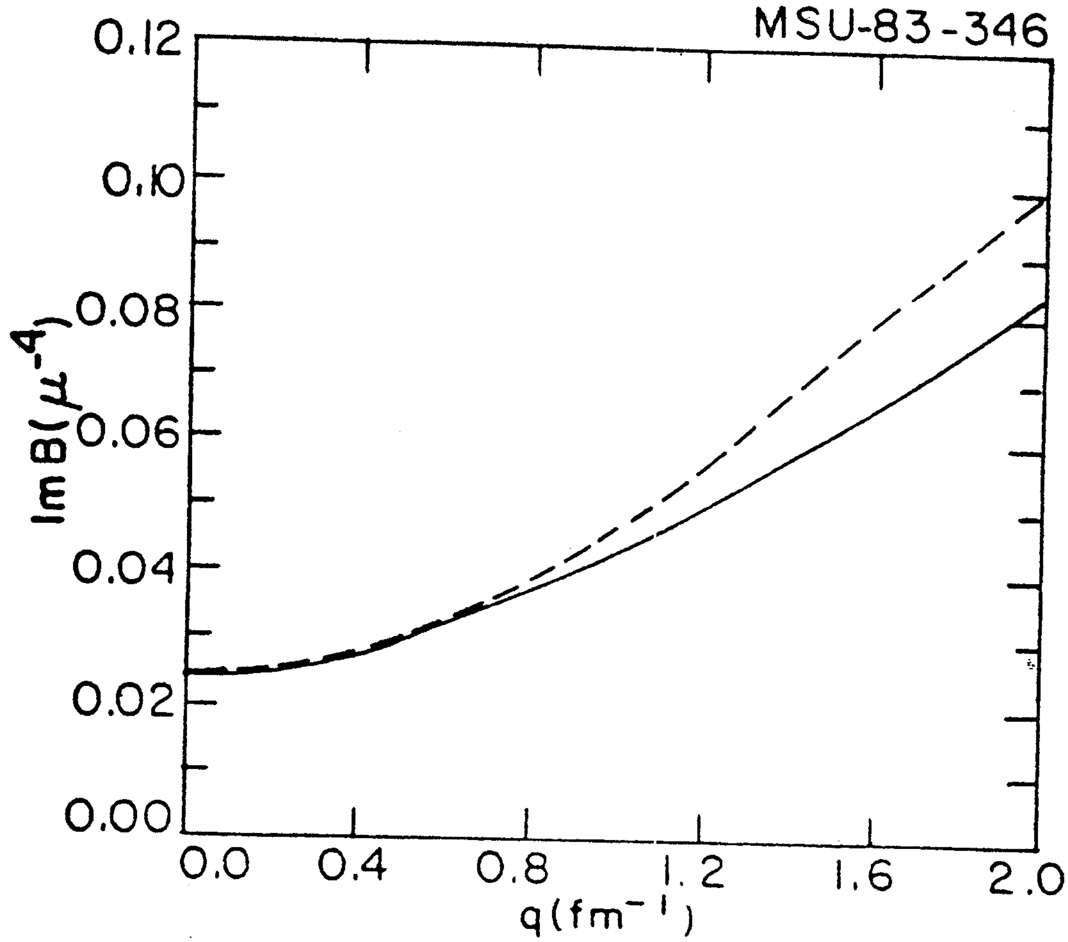

Figure III.2

The imaginary component of the S wave pion absorption parameter of nuclear matter vs. pion momentum.
Dashed line corresponds to a rescattered pion energy of 0.5 ω .
Solid line corresponds to a rescattered pion energy of 0.5 μ .

We try to improve the result [Ref. III.8] . By the same method as used in Chapter II we introduce the effect of the nuclear medium on the pion propagator.

Also the medium affects the absorption vertex, so that it is renormalized [Ref. III.8] .

These modifications can be taken into account by simply replacing the Yukawa one pion exchange potential i.e.

$$Y_1(\mu^* r) = \left(1 + \frac{1}{\mu^* r}\right) \exp(-\mu^* r)/\mu^* r \qquad (3\text{-}1\text{-}8)$$

by the following integral

$$Y_1(\mu^* r) \longrightarrow \frac{2}{\pi \mu^{*2}} \int_0^\infty \frac{k^3 J_1(kr)\Gamma(k) f_\pi^2(k)}{k^2 + \frac{3}{4}\mu^2 + \Pi(k)} dk \qquad (3\text{-}1\text{-}9)$$

where the vertex renormalization is given by

$$\Gamma(k) = 1 \Big/ \left(1 - \frac{\lambda}{3} \Pi_0(k) / k^2 f_\pi^2(k)\right) \qquad (3\text{-}1\text{-}10)$$

In both of the above equations the pion's effective mass

is $\mu^* = \sqrt{\mu^2 - \omega'^2}$. The integrand of Equation (3-1-9) contains a regular pion monopole form factor given by Equation (2-4-5) .

Based on these modifications for threshold S wave pions, the variation of the absorption parameter Im B_o is studied as a function of the LLEE parameter i.e. λ . The results are shown in Figure III.3 . In this figure it is quite interesting to note that by choosing λ = 1.6 - 1.8 which is exactly in the region of the preferred range of λ [Ref. III.9] , we get the proper enhancement which we need to match the pionic atom data. Also this result agrees with the absorption parameter which is used in phenomenological πA scattering optical potentials.

A detailed study of the pion's self-energy reveals the possibilities of an interesting physical phenomenon. To make the point clear, one should recall the regular static OPE potential between two nucleons in vacuum, i.e.

$$V_\pi(q) = -\left(f_{\pi NN}/\mu\right)^2 \frac{(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})}{q^2+\mu^2} \qquad (3\text{-}1\text{-}11)$$

where the coupling constant is given in [Ref. II.9] and $\vec{q}$ is the pion's momentum transfer. Replacing the vacuum by a Fermi sea, the pion polarizes the medium. Polarization of the medium can be viewed as the virtual excitation of nucleon-hole as well as isobar-hole pairs carrying the pion's quantum numbers, shown in Figure III.4

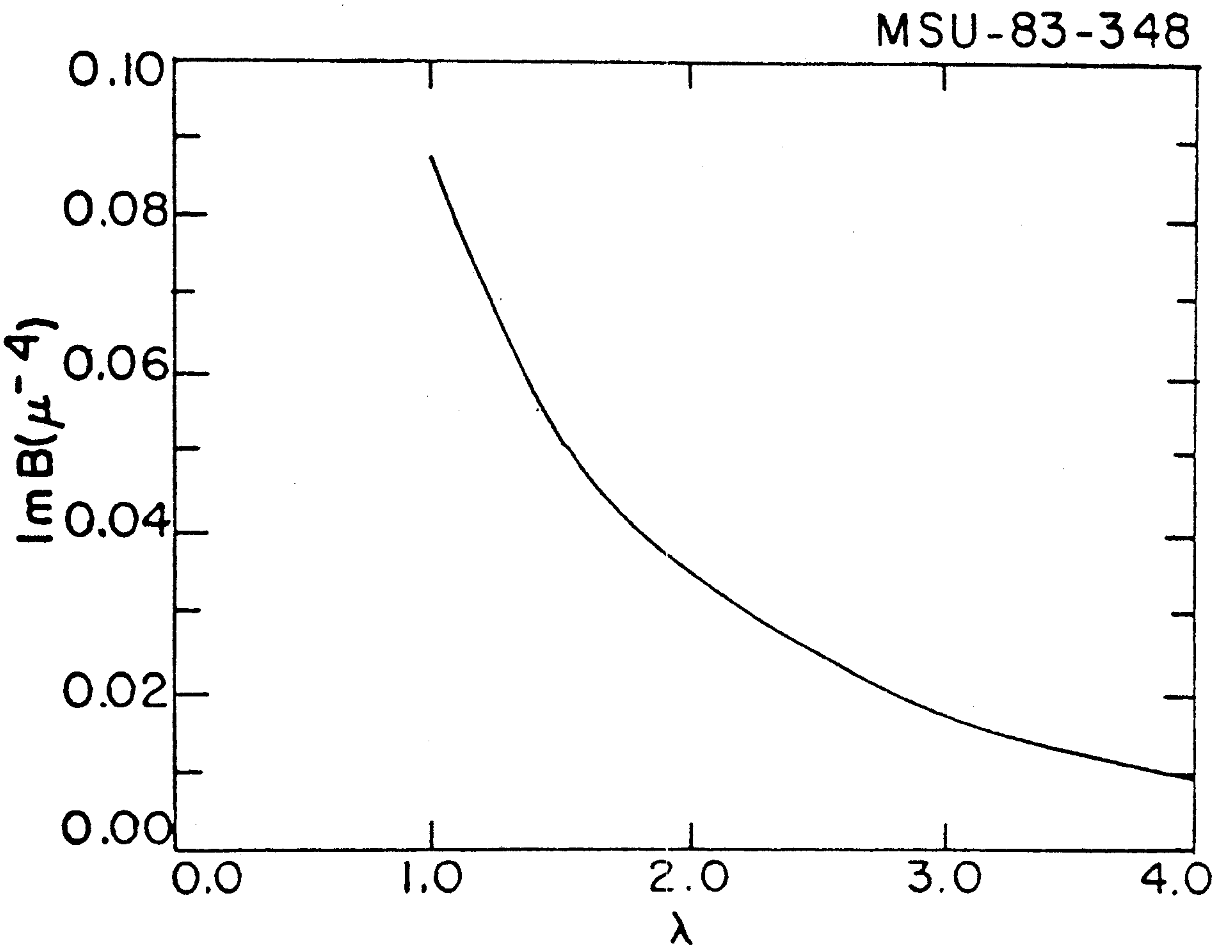

Figure III.3

The imaginary component of the two-body S wave threshold pion absorption parameter of nuclear matter vs. LLEE parameter λ .

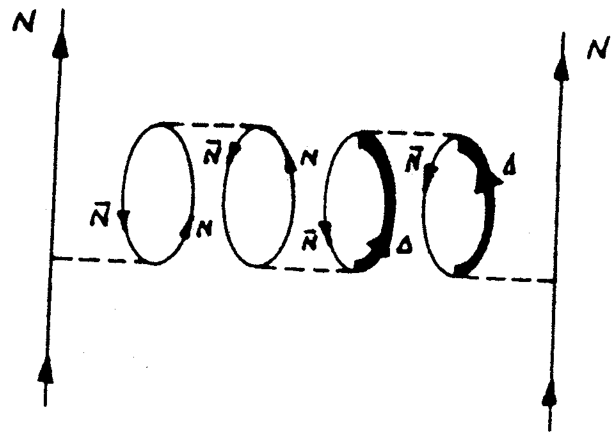

Figure III.4 Nuclear medium polarization due to pion propagation

The polarization is determined by the strength of the P wave pion-nucleon coupling, the density ρ of the medium and by the strength of the short-range correlation accompanying the driven OPE interaction. The modification can then be written as

$$\tilde{V}_{\pi}(q) = V_{\pi}(q) / \epsilon(q) \qquad (3\text{-}1\text{-}12)$$

where the mesonic polarization effects are summarized in terms of the diamesic function $\epsilon(q)$. If, in a much simplified picture, the repulsive short range correlations are parametrized by $g' \, (\vec{\sigma}^{1} \cdot \vec{\sigma}^{2})(\vec{\tau}^{1} \cdot \vec{\tau}^{2}) \, \delta(r)$ where g' is related to the LLEE parameter λ introduced in Chapter II, g' = $\lambda/3$

$$\epsilon(q) = 1 + \left[g' - (f_{\pi NN}/\mu)^2 \frac{q^2}{q^2+\mu^2} \right] \chi(q,\rho) \qquad (3\text{-}1\text{-}13)$$

where $\chi(q,\rho)$ contains the information about each individual nucleon-hole or isobar-hole excitation, represented by a single bubble in Figure III.4 .

The size of the diamesic function is essentially determined by the competition between the attraction from OPE and the short-range repulsion controlled by g' .

$1/\epsilon(q)$ becomes maximum, at about $q \cong 3\mu$ and for g' = 0.4, and at regular nuclear matter density, $\rho \cong 0.17$ fm^{-3}, it becomes infinite. The singularity of $\epsilon(q)$ at $q \cong 3\mu$ comes from the strong amplification of the pion's field.

This amplification corresponds to the existance of a large number of pions, and nuclei should then undergo a phase transition. Nuclear structure would be reordered such that the ground state would have a strong pion-field component and long-range ordering of the nucleon spins would occur due to the spin dependence of the pion-nucleon interaction. However the empirical value of g' is = 0.6 - 0.8, corresponding to $\lambda \cong 1.8 - 2.4$ and condensation is avoided at nuclear matter density.

A nucleus might, however, undergo a phase transition at higher density [Ref. III.10].

In heavy ion collisions, where creation of exotic objects with densities higher than regular nuclear densities are possible experimentally one might see the phase transition phenomena.

In Figure III.5 we have shown the behavior of Im B_0 with inclusion of the medium correction at λ = 1.5 vs. pion lab momentum. This shows the overall many-body effect in S wave pion absorption. The curve is plotted for $\omega' = 1/2\mu$. By comparing with Figure III.2 one sees that not only does the many-body effect enhance the threshold value of Im B_0, but it gives less variation with energy.

In Figure III.6 the effects of the medium correction at λ = 1.5 on the dispersive component Re B_0 of the aborption parameter is shown. There is no noticeable amplification at low energies.

To conclude the discusion of the S wave pion absorption in nuclear matter, from a two-body mechanism point of view, we should talk about the ratio of pion absorption in two different possible initial isospin channels. To be more specific, for instance one could think of π^- absorption on initial np or pp nucleon pairs.

The interesting quantity is

$$R_s = \frac{(\pi^- np \rightarrow nn)}{(\pi^- pp \rightarrow np)} \tag{3-1-14}$$

The observed ratio for the above quasifree processes is about 10 . The most sophisticated calculation based on the two-body absorption mechanism [Ref. III.6] predicts R_s = 3 . The same work [Ref. III.6] predicts Im $B_0 = 0.036\mu^{-4}$

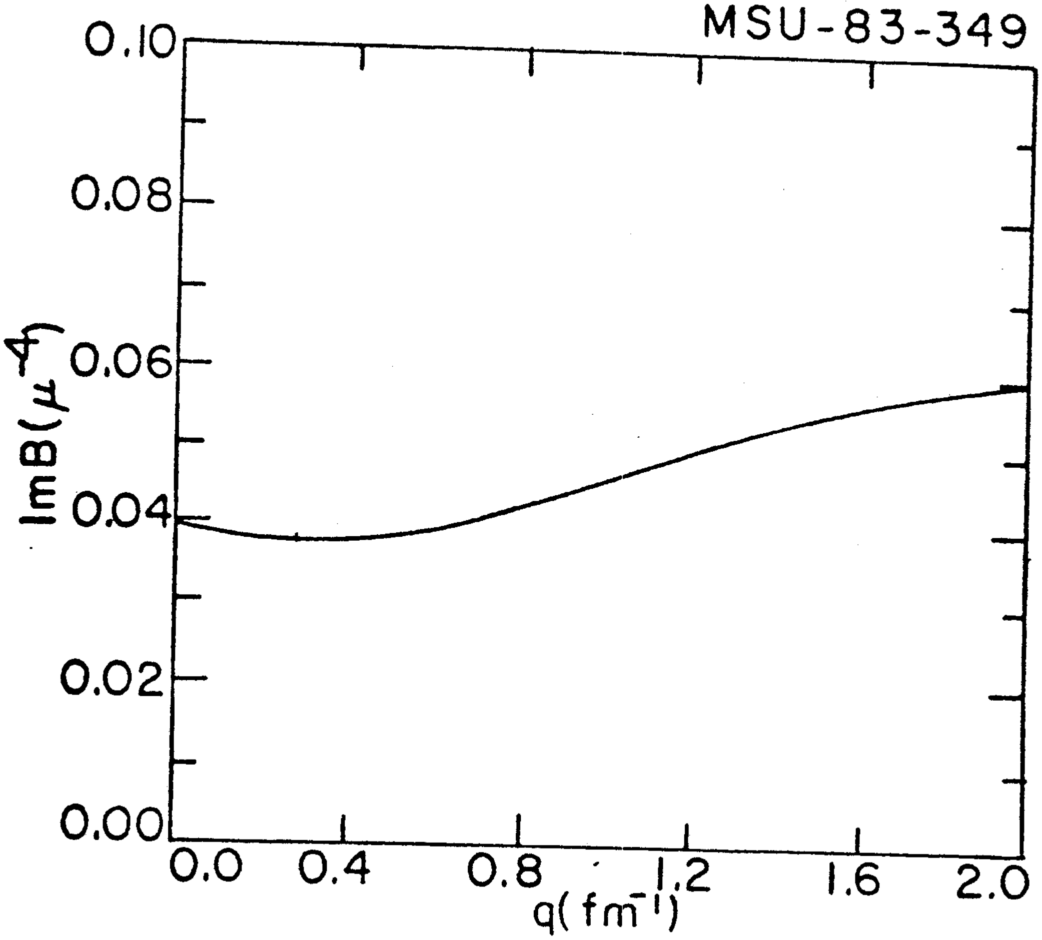

Figure III.5

The imaginary component of the two-body S wave absorption parameter of nuclear matter with inclusion of medium effects for λ = 1.5 vs. pion momentum.

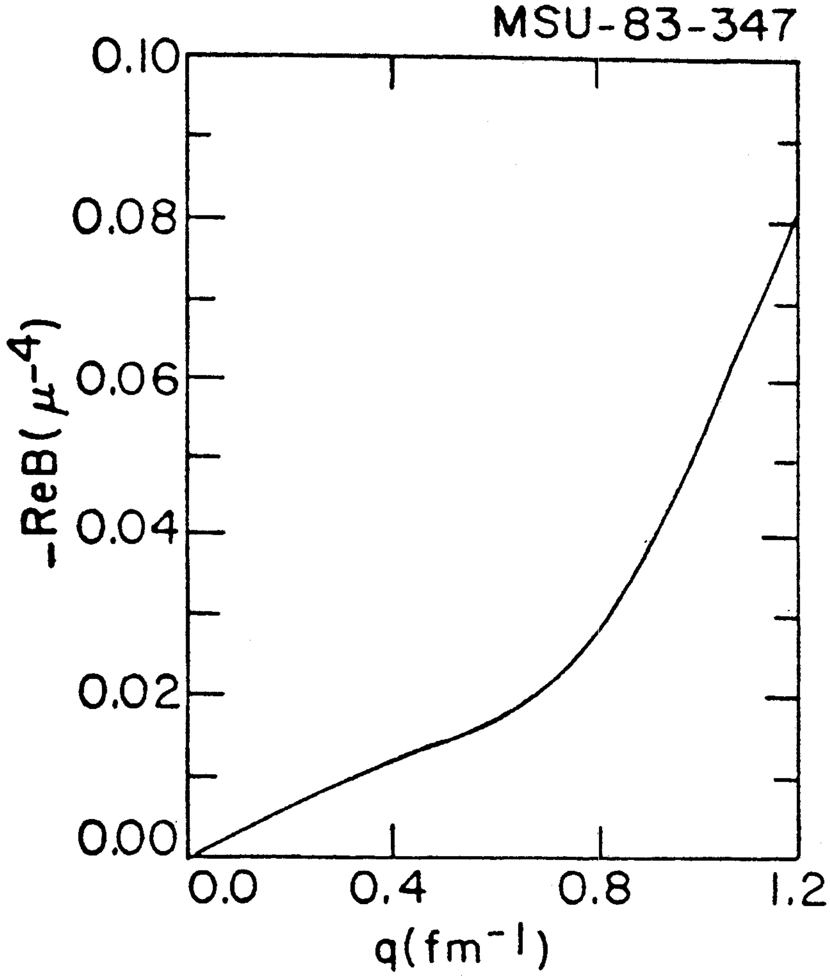

Figure III.6

The dispersive component of the S wave pion absorption parameter of nuclear matter with resct-ted pion energy of $0.5\,\mu$ and medium effects for $\lambda = 1.5$.

which is close to the parameter used to fit pionic atom level widths [Ref. III.7] . Therefore [Ref.III.11] , one conclusion could be the involvement of more than two nucleons in the absorption process.

3-2 S Wave Pion Absorption in Finite Nuclei

Motivated by these results, we have studied the effects of the pion self energy in S wave pion absorption in finite nuclei. Here, harmonic oscillator functions were used to construct the initial nucleon pair wave function and plane waves were used for knockout nucleons.

Since the medium effect is calculated for nuclear matter, the absorption rate is averaged over the nuclear density, i.e.

$$\Gamma = \int \rho(r)\,\Gamma(r)\,d\vec{r} \qquad (3\text{-}2\text{-}1)$$

To be consistent we set $\lambda = 1.5$ and for convenience, instead of the absorption parameter we studied the absorption cross-section.

The absorption cross-section is

$$\sigma = 2\pi\mu/k \sum_{fi} |T_{fi}|^2\, \delta(\epsilon_i + \omega - \epsilon_f) \qquad (3\text{-}2\text{-}2)$$

On the other hand the imaginary component of the scattering length is

$$Im\, a = \frac{\mu \Gamma}{4\pi} \tag{3-2-3}$$

where the absorption rate Γ is

$$\Gamma = 2\pi \sum_{fi} | T_{fi} |^2 \, \delta(E_f + \omega - E_i) \tag{3-2-4}$$

By combining the above three equations we get

$$\sigma \doteq \frac{4\pi}{K} Im\, a \tag{3-2-5}$$

Hitherto we have used for the vertex form factors Equation (3-2-5) the standard value Λ = 1200 Mev , obtained from fits to π- deuteron absorption. At this point we investigate the dependence of the S wave absorption both on medium effects and the vertex form factor. The results are shown in Figures III.7 and III.8 , which show the absorption cross-section and the imaginary part of the S wave pion scattering length for ^{4}He as a function of the pion incident momentum, for different choices of the vertex form factor parameter Λ, and with and without medium effects. As in Chapter II, for calculating medium effects we take λ = 1.5.

Referring to Figure III.7, absorption cross-section versus pion momentum, we have 2 curves. The top curve,

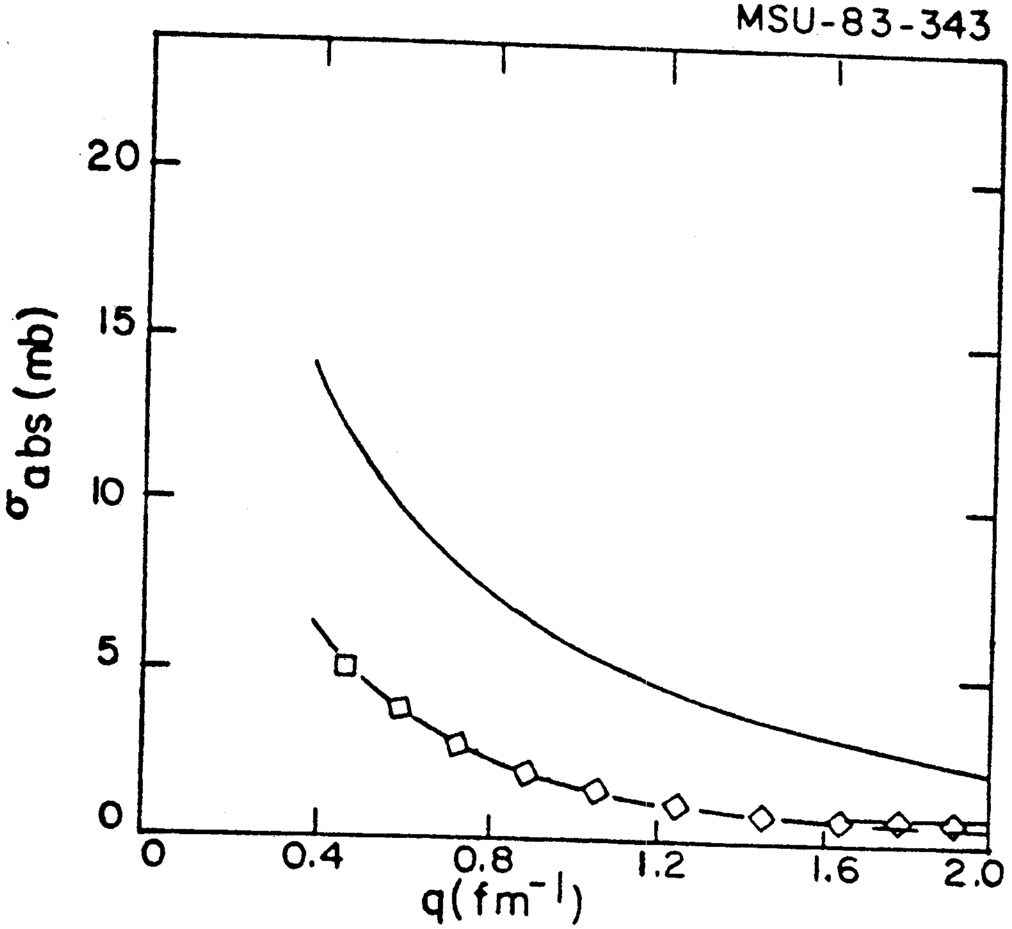

Figure III.7

S wave pion absorption cros-section of ^{4}He vs. pion momentum.
The solid line on the scale of this drawing corresponds to the following three almost indistinguishable cases
one π exchange with Λ_π = 1200 Mev and no medeium effects
one π exchange with $\Lambda_\pi = \infty$ and medium effects at λ = 1.5
one π exchange with Λ_π = 1200 Mev and medium effects at λ = 1.5
Diamonds correspond to one pion exchange with Λ_π = 700 Mev and with and without medium effects at λ = 1.5 .

solid line, corresponds to the standard case Λ = 1200 Mev, no medium correction and the bottom case, emphasized by diamonds, corresponds to Λ = 700 Mev, no medium correction. Thus we see that the absorption cross-section is extremely sensitive to the vertex parameter Λ , i.e. the size of the source, $\langle r^2 \rangle^{1/2} = \sqrt{6}/\Lambda$.

Actually the top curve, solid line, also corresponds to the standard case Λ = 1200 Mev, with medium correction calculated for λ = 1.5 , which is indistinguishable on the scale of the drawing from the case without medium correction. The same is true for the bottom curve. Thus also for Λ = 700 Mev, the effect of the medium correction, for λ = 1.5, is neglegible. Thus medium correction effects for a small finite nucleus, i.e. ^{4}He , do not lead to an amplification of the absorption, unlike the case of nuclear matter, where medium effects gave an amplification factor $\cong$ 2. The difference between the two cases can be explained as the low effective density given by the folding procedure (3-2-1) (surface absorption).

Figure III.8 shows the same result for the imaginary part of the scattering length. The case of Λ = 1200 Mev, no medium effect, solid line, and Λ = 1200 Mev, medium effect for λ = 1.5, dashed-dot line, are just distinguishable from one another on this scale. The experimental value at zero energy is also shown. It agrees with the atandard case Λ = 1200 mev, i.e. source radius $\cong$ 0.4 fm, with or without medium effect. It is clear that Λ = 700 Mev, i.e.

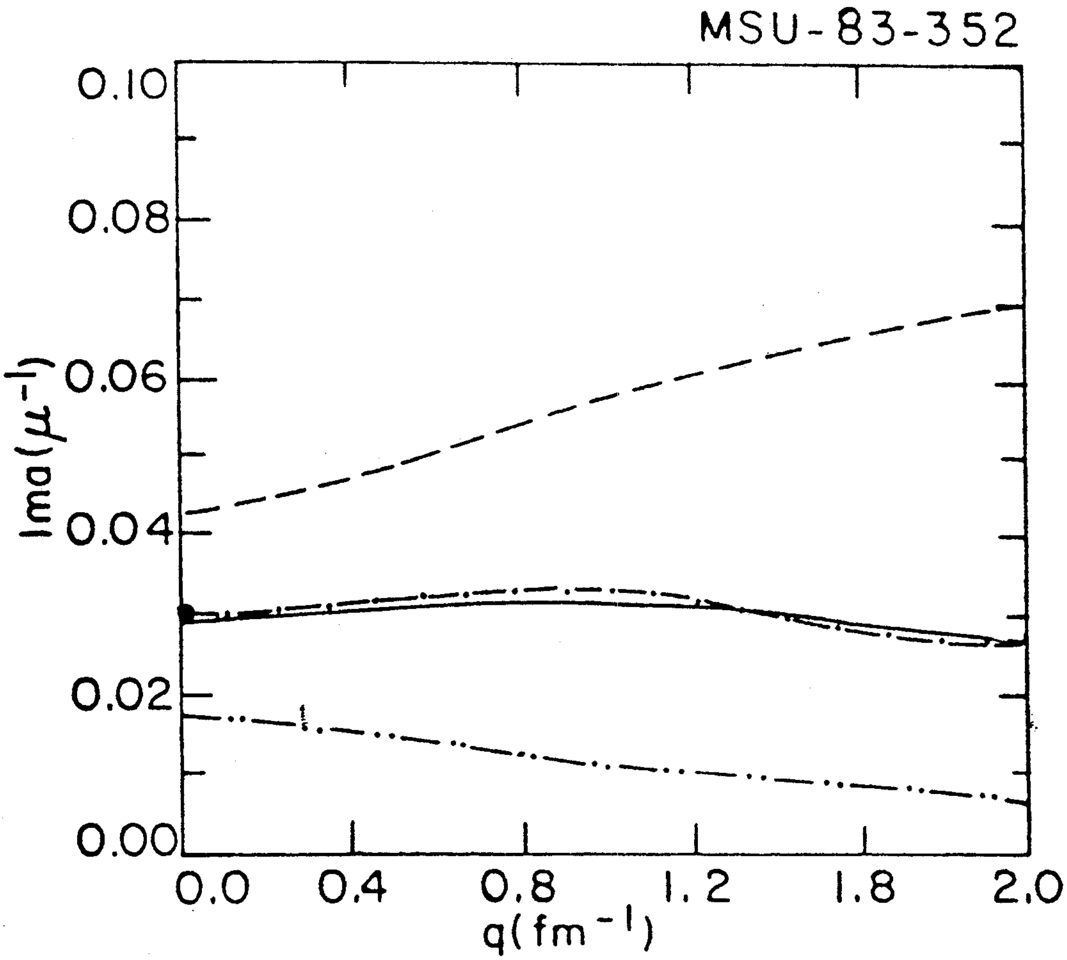

Figure III.8

The imaginary component of the S wave pion scattering length of ^4He vs. pion momentum.
Dashed line corresponds to one π exchange with $\Lambda_\pi = \infty$ and no medium effects.
Dashed-dot line corresponds to one π exchange with Λ_π = 1200 Mev and medium effects for λ = 1.5 .
Solid line corresponds to one π exchange with Λ_π = 1200 Mev and no medium effects.
Dashed-dot-dot line corresponds to one π exchange with $\Lambda\pi$ = 700 Mev with and without medium effects for λ = 1.5 .
• is the experimental value for threshold pion [Ref. III.7] .

source radius $\cong 0.7$ fm, gives a result that is much too small to agree with experiment.

Referring back to Figure III.7 we notice that the S wave pion absorption cross-section falls smoothly with increasing pion momentum, i.e. shows a typical $1/v$ behavior. So since the cross-section has an appreciable magnitude, $\gtrsim$ 15 mb, only up to incident pion energies of $\cong$ 20 Mev, S wave pion absorption is confined to low energies. P wave rescattering absorption quickly becomes the dominant mode, and will be discussed in the next chapter.

3-3 On The Angular Distribution of (π , 2N)

Experimental data for low energy pions indicates that knocked out nucleons are emitted back to back and have an isotropic angular distribution. In this section we study the angular dependence of the scattering cross- section for a typical (π , 2N) reaction, assuming S wave absorption only.

The calculation is carried out for ^{4}He . For an incident π^+ beam, we have the following four different absorption possibilities,

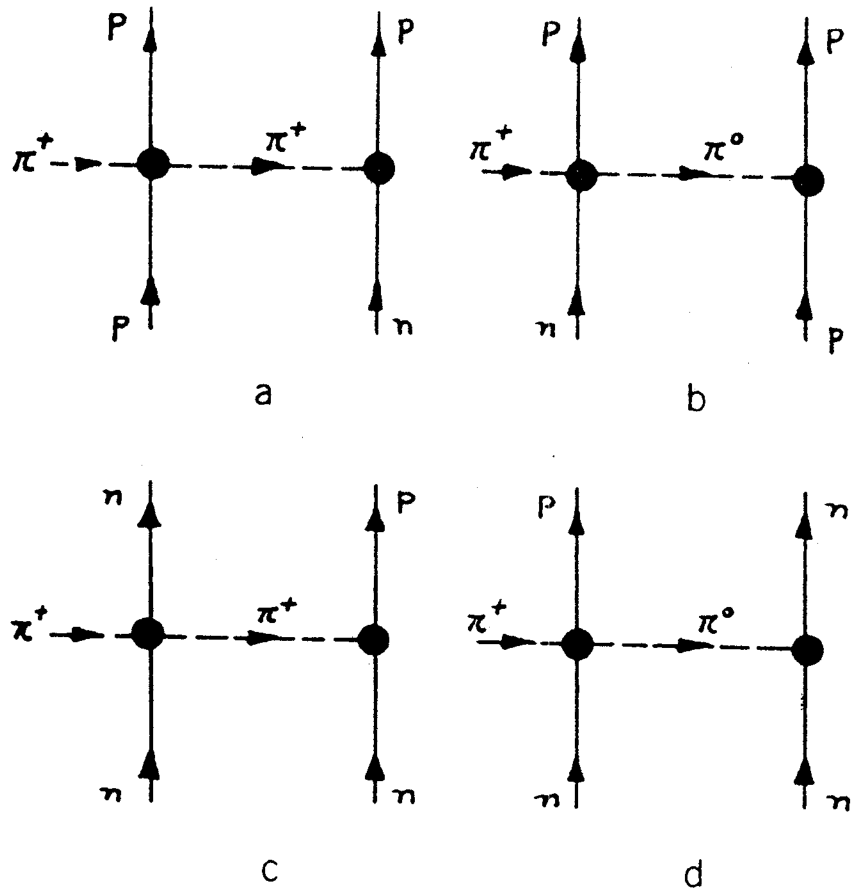

Figure III.9 The four possible S wave π^+ rescattering absorption mechanism

The scattering cross section is

$$\frac{d\sigma}{d\Omega} = 2\pi \, |T_{fi}|^2 \tag{3-3-1}$$

where T_{fi} is the matrix element of the transition (absorption) operator between the initial and final nucleon pair states. The operator is given by Equation (3-1-7). Referring to Figure III.1 the symmetrized amplitude can be rewritten in the following form

$$T = (\tau^1 + \tau^2)_+ t^1 + (\tau^1 - \tau^2)_+ t^2 - i(\vec{\tau}^1 \times \vec{\tau}^2)_+ t^3 \qquad (3\text{-}3\text{-}2)$$

where the τ^1 and τ^2 are the usual isospin operators of the first and second nucleon respectively and (+) stands for the spherical component of the given isospin operator

$$t^{1,2} = f_{\pi NN}/\mu^2 \lambda_1 \mu^{*2}/\sqrt{\omega}\, Y_1(\mu^* r)\, e^{i\vec{q}\cdot\vec{R}} \Big[\cos(\tfrac{1}{2}\vec{q}\cdot\vec{r})(\vec{\sigma}^1 \mp \vec{\sigma}^2)\cdot\hat{r} - i\sin(\tfrac{1}{2}\vec{q}\cdot\vec{r})(\vec{\sigma}^1 \pm \vec{\sigma}^2)\cdot\hat{r} \Big] \qquad (3\text{-}3\text{-}3a)$$

$$t^3 = - f_{\pi NN}/\mu^3 \lambda_2 \mu^{*2}/\sqrt{\omega}\, Y_1(\mu^* r)\, e^{i\vec{q}\cdot\vec{R}} (\omega + \omega') \Big[\cos(\tfrac{1}{2}\vec{q}\cdot\vec{r})(\vec{\sigma}^1 + \vec{\sigma}^2)\cdot\hat{r} - i\sin(\tfrac{1}{2}\vec{q}\cdot\vec{r})(\vec{\sigma}^1 - \vec{\sigma}^2)\cdot\hat{r} \Big] \qquad (3\text{-}3\text{-}3b)$$

Here σ^1 and σ^2 are the usual Pauli spin operators of the first and second nucleon respectively, $\vec{q}$ is the incident pion momentum. The scattering lengths λ_1 and λ_2 are given in Chapter II, the coupling constant $f_{\pi NN}$ and the pion effective mass μ^* are given in [Ref. III.5]. The modified Yukawa function $Y_1(\mu^* r)$ is given by Equation

(3-1-8). The CM coordinate of the two nucleons is $\vec{R} = 1/2(\vec{r}_1 + \vec{r}_2)$ and the relative separation of two nucleons $\vec{r} = \vec{r}_1 - \vec{r}_2$.

Since we are interested in low energy pions, the above amplitude can be approximated. This approximation is a trivial one, that is

$$\sin(\tfrac{1}{2}\vec{q}\cdot\vec{r}) = 0 \quad ; \quad \exp(i\vec{q}\cdot\vec{R}) \simeq 1 \tag{3-3-4}$$

As the pion brings in no appreciable momentum, the center of mass motion remains unchanged so we only have to consider the relative wave function of the two nucleons. Equations (3-3-3a) and (3-3-3b) become

$$t^{\frac{1}{2}} = f_{\pi NN}/\mu^2 \, \lambda_1 \, \mu^{*2}/\sqrt{\omega} \; Y_1(\mu^* r)\,(\vec{\sigma}^1 \mp \vec{\sigma}^2)\cdot\hat{r}\,\cos(\tfrac{1}{2}\vec{q}\cdot\vec{r}) \tag{3-3-5a}$$

$$t^3 = f_{\pi NN}/\mu^3 \, \lambda_2 \, \mu^{*2}/\sqrt{\omega} \; Y_1(\mu^* r)(\omega+\omega')(\vec{\sigma}^1 + \vec{\sigma}^2)\cdot\hat{r}\,\cos(\tfrac{1}{2}\vec{q}\cdot\vec{r}) \tag{3-3-5b}$$

To calculate T_{fi} , it is more convenient to label the initial and final nucleon pair states in terms of total isospin T and spin S quantum numbers. That is $|i\rangle = |T_i, S_i\rangle$, and $|f\rangle = |T_f, S_f\rangle$.

Hence for π^+(pn, pp) i.e. Figure III.9a,b non-vanishing spin isospin matrix elements become

$$T_{fi} \simeq \langle pp, {T_f=1 \atop S_f=1} | (\tau^1-\tau^2)_+ (\vec{\sigma}^1+\vec{\sigma}^2) | pn, {T_i=0 \atop S_i=1} \rangle - \tag{3-3-6}$$

$$\langle pp, {T_f=1 \atop S_f=1} | i(\vec{\tau}^1 \times \vec{\tau}^2)_+ (\vec{\sigma}^1+\vec{\sigma}^2) | pn, {T_i=0 \atop S_i=1} \rangle +$$

$$\langle pp, {T_f=1 \atop S_f=1} | (\tau^1+\tau^2)_+ (\vec{\sigma}^1-\vec{\sigma}^2) | pn, {T_i=1 \atop S_i=0} \rangle$$

and for π^+(nn, pn) i.e. Figure III. 9c,d the non vanishing spin isospin matrix element becomes

$$T_{fi} = \langle pn, {T_f=1 \atop S_f=1} | (\tau^1+\tau^2)_+ (\vec{\sigma}^1-\vec{\sigma}^2) | nn, {T_i=1 \atop S_i=0} \rangle \tag{3-3-7}$$

In this special channel only one term survives so we look at this case.

The complete form of Equation (3-3-7) is

$$T_{fi} \simeq \langle pn, {T_f=1 \atop S_f=1} | (\tau^1+\tau^2)_+ t^1 | nn, {T_i=1 \atop S_i=0} \rangle \tag{3-3-8}$$

Since the initial nn pair is in the singlet spin and triplet isospin state, it's spatial wave function is even. We confine ourselves to orbital angular momentum $L = 0$.

A harmonic oscillator basis is used to construct the spatial wave function, i.e.

$$\psi_i(\vec{r}) = R_{n\ell}(r)\, Y_{\ell m}(\hat{r}) \tag{3-3-9}$$

where n = l = m = 0.

We use plane waves to describe the spatial components of the final nucleon states and since the final pn pair is coupled to an isospin and spin triplet state, the anti-symmetrized spatial wave function is

$$\psi_f(\vec{r}) = \sum_{l_f} (i)^{l_f} j_{l_f}(k_f r)\, Y^*_{l_f m_f}(\hat{r})\, Y_{l_f m_f}(\hat{k}_f) \qquad (3\text{-}3\text{-}10)$$

where $\vec{k}_f$ is the relative momentum of the two ejected nucleons, shown in Figure III.10 .

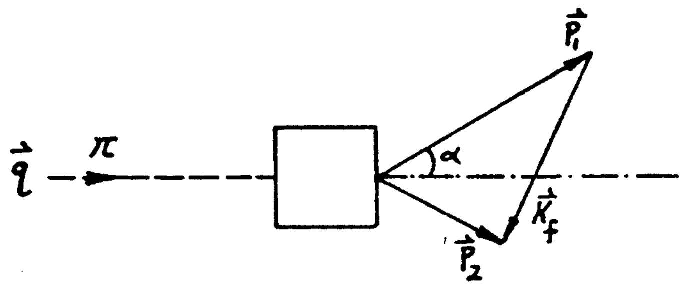

Figure III.10 Schematic (π , 2N) reaction

$\cos(\frac{1}{2}\vec{q}\cdot\vec{r})$ is expanded in terms of the partial waves

$$\cos(\tfrac{1}{2}\vec{q}\cdot\vec{r}) = 4\pi \sum_{l}^{(+)} (i)^{l}\, j_l(\tfrac{1}{2}qr)\, Y^*_{lm}(\hat{r})\, Y_{lm}(\hat{q}) \qquad (3\text{-}3\text{-}11)$$

where (+) means only even values for l are allowed. By setting l = 0 in Equation (3-3-11) , appropriate for low energy pions, conservation of angular momentum confines the final angular momentum in Equation (3-3-10) to l = 1 .

With these approximations the scattering cross section i.e. Equation (3-3-1) becomes

$$\frac{d\sigma}{d\Omega} \simeq \frac{1}{\omega} I^2(\mu^*, k_f) \, |Y_{lm}(\hat{k}_f)|^2 \qquad (3\text{-}3\text{-}12)$$

where the radial transition matrix element I is defined as

$$I(\mu^*, k_f) = \int_0^\infty j_1(k_f r)\, j_0(\tfrac{1}{2} q r)\, y_1(\mu^* r)\, R_{00}(r)\, r^2 dr \qquad (3\text{-}3\text{-}13)$$

here $R_{00} = \exp(-\frac{1}{2}\nu r^2)$ and for He we used $\nu = 0.493\ \mathrm{fm}^{-2}$.

The polarization information of the ejected paired nucleons is carried by $Y_{lm}(\hat{k}_f)$. Therefore, by summing over all the possible polarization directions we get

$$\frac{d\sigma}{d\Omega} = N \frac{1}{\omega} |I(\mu^*, k_f)|^2 \qquad (3\text{-}3\text{-}14)$$

where the N contains all the energy independent factors.

Any anisotropy in scattering cross section i.e. Equation (3-3-14) is contained in the radial matrix element I , which depends on k_f . But conservation of energy and momentum give no dependance of k_f on α and the emission is isotropic.

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Chapter IV

P WAVE PION ABSORPTION

4-1 In this section we concentrate our study on P wave pion absorption in a nucleus.

Free πN total scattering cross sections show a resonance peak at about 200 Mev pion lab energy, with an approximate width of 110 - 120 Mev. This well established resonance is interpreted as the formation of an unstable intermediate resonance state in spin and isospin 3/2 channel i.e. the Δ resonance.

Based on this fact, P wave pion should be absorbed by a cluster of at least two nucleons by the process

$$\pi + N_1 \longrightarrow \Delta \quad \& \quad \Delta + N_2 \longrightarrow N_1 + N_2$$

To indicate the two-body absorption mechanism the optical potential is parameterized in the following form [Ref. IV.1]

$$2\omega U_{opt} = -4\pi \vec{\nabla} C_0 \rho^2 \vec{\nabla} \qquad (4\text{-}1\text{-}1)$$

where C_0 is the P wave pion absorption parameter. The imaginary component of C_0 is related to the absorption rate Γ by Equation (3-2-4) via [Ref. IV.2]

$$Im\, C_0 = \frac{\omega}{4\pi k^2 \rho^2 \Omega} \Gamma \qquad (4\text{-}1\text{-}2)$$

where K is the incident pion's momentum and Ω is the nuclear volume.

Calculation of the absorption rate per unit volume i.e. Γ/Ω is based on the absorption amplitude constructed from the following two diagrams, Figure IV.1

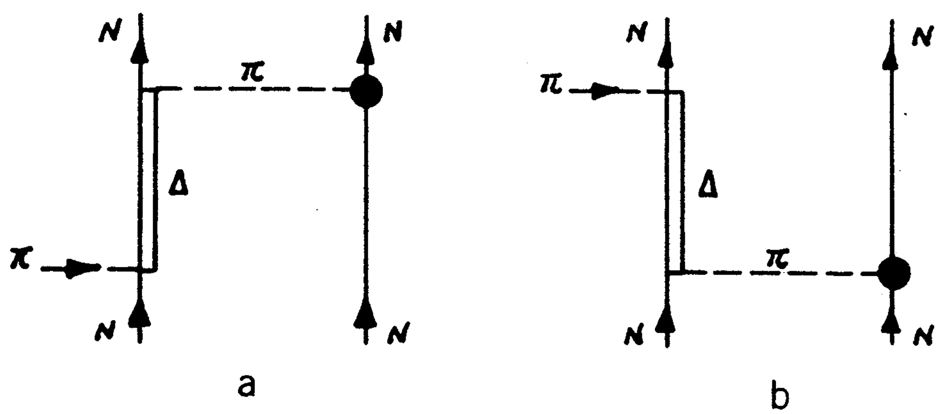

Figure IV.1 The direct and crossed channels of the P wave pion rescattering absorption mechanism with intermediate one π exchange

where Figure IV.1a,b are refered to as the direct and crossed channels.

The direct channel has an obvious physical meaning, the crossed channel is interpreted as the destruction by an incident real pion of an intermediate Δ which is formed already in the target nucleus by exchange of a virtual meson.

The formation of the intermediate Δ resonance on one of the nucleons has a direct effect on the energy dependence of the absorption. Im C_0 vs. pion lab energy has a resonance shape.

The Δ decays not only into πN , ($\Delta \rightarrow \pi N$) but also into ρN , ($\Delta \rightarrow \rho N$) . Both of these two modes should be included in the construction of the absorption amplitude. So the following processes are added, Figure IV.2

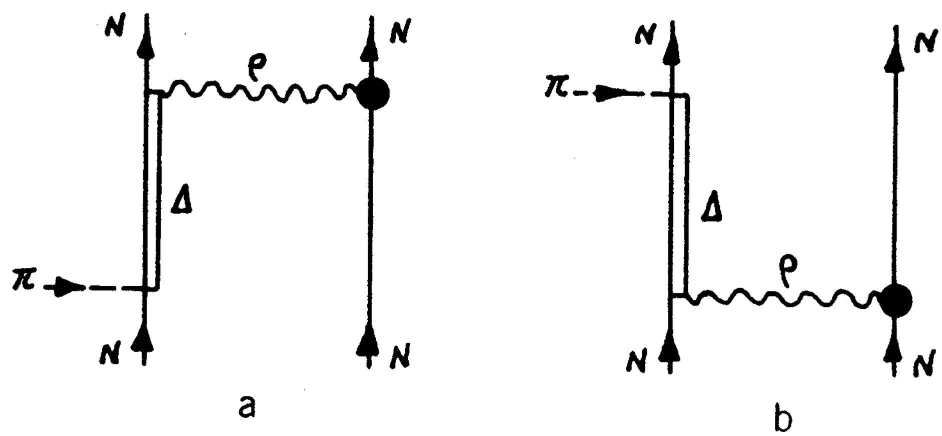

Figure IV.2 The direct and crossed channels of the P wave pion rescattering absorption mechanism with intermediate ρ exchange

To construct the absorption amplitude in addition to Equation (2-1-1), we use the Hamiltonian densities,

$$H_{\pi N \Delta} = f_\Delta / \mu \ \vec{\chi}^{\dagger} \vec{\xi}^{\dagger} \cdot \vec{\nabla} \vec{\phi} \ \xi \ \chi + h.c. \qquad (4\text{-}1\text{-}3a)$$

$$H_{\rho N\Delta} = g_{\rho\Delta}/2m \; \chi^{+} \xi^{+} \cdot (\vec{\nabla} \times \vec{\rho}) \xi \chi + \text{h.c.} \tag{4-1-3b}$$

$$H_{\rho NN} = g_{\rho}/2m \, (1+\varkappa) \, \chi^{+} \xi^{+} (\vec{\sigma} \times \vec{\nabla}) \vec{\rho} \cdot \vec{\tau} \, \xi \chi \tag{4-1-3c}$$

where $g_\rho^2/4\pi = 0.55$, $\varkappa = 6.6$ and $g_{\rho\Delta} = 6\sqrt{2}/5 \; g_\rho(1+\varkappa)$ as predicted in the static quark model [Ref. IV.3] with $\pi N\Delta$ coupling constant $f_\Delta^2/4\pi = 0.32$.

Here ξ and χ are the nucleon spinor and isospinor and $\vec{\xi}$ and $\vec{\chi}$ are the Δ vector spinor and vector isospinor. The pion isovector field operator is $\vec{\phi}$ and the rho vector meson isovector field operator is $\vec{\rho}$, the mass of the pion and nucleon are denoted μ and m respectively.

By applying the standard Feynman rules for Figure IV.1 the absorption amplitude becomes

$$T = i \, \frac{8}{9} \, \frac{\sqrt{2}}{\sqrt{2\omega}} \, f_{\pi NN}/\mu \, (f_\Delta/\mu)^2 \, \frac{\vec{\sigma}_2 \cdot \vec{k}_2}{D(k_2^2+\mu^2-\omega'^2)} \Big[\vec{k}_2 \, \tau_+^2 - \frac{1}{4} (\vec{\sigma}_1 \times \vec{k}_2)(\vec{\tau}^1 \times \vec{\tau}^2)_+ \Big] \cdot \vec{k} \tag{4-1-4}$$

where K_2 and ω' are the momentum and energy of the rescattered pion. The (+) component of isospin operator τ_+^2 is defined $\tau_+^2 = \frac{1}{2}(\tau_x^2 + i\tau_y^2)$ with an analogous definition for that of the $(\vec{\tau}' \times \vec{\tau}^2)_+$.

The energy denominator D is

$$D = \omega_R - \omega - \frac{i}{2}\Gamma_\Delta \qquad (4\text{-}1\text{-}5)$$

where

$$\omega_R = m_\Delta - m + k^2/2m_\Delta \qquad (4\text{-}1\text{-}6)$$

here m_Δ is the Δ mass, and the Δ's width is given by Equation (2-4-11) .

For ρ meson exchange i.e. Figure IV.2a the absorption amplitude becomes

$$T_\rho = -i\,\frac{2}{9}\,\frac{\sqrt{2}}{\sqrt{2\omega}}\,\frac{g_\rho}{m^2\mu}\,g_{\rho\Delta}\,\frac{1+\kappa}{D}\,f_\Delta\,\frac{1}{k_2^2+m_\rho^2-\omega'^2}\Big[\qquad (4\text{-}1\text{-}7)$$
$$\vec{\sigma}_2\cdot\vec{k}_2\left[\vec{k}_2\,\tau_+^2 - \frac{1}{4}(\vec{\sigma}_1\times\vec{k}_2)(\vec{\tau}^{\,1}\times\vec{\tau}^{\,2})_+\right] -$$
$$k_2^2\left[\vec{\sigma}_2\,\tau_+^2 - \frac{1}{4}(\vec{\sigma}^{\,1}\times\vec{\sigma}^{\,2})(\vec{\tau}^{\,1}\times\vec{\tau}^{\,2})_+\right]\Big]\cdot\vec{k}$$

Spin and isospin matrix element calculations of these operators between initial and final paired nucleons are quite complicated. The results are given in [Ref. IV.4] . However, for π and ρ mesons, there are the following relations

$$\xi_\pi Y_0(\mu^* r) \longrightarrow 2\,\xi_\rho Y_0(m_\rho^* r) \qquad (4\text{-}1\text{-}8a)$$

$$\xi_\pi Y_2(\mu^* r) \longrightarrow -\,\xi_\rho Y_2(m_\rho^* r) \qquad (4\text{-}1\text{-}8b)$$

where ξ_π and ξ_ρ contain all the kinematical factors given in [Ref. IV.4] and the effective masses of pion and rho mesons are given by $\mu^* = \sqrt{\mu^2 - \omega'^2}$ and $m^*_\rho = \sqrt{m_\rho^2 - \omega'^2}$. Thus ρ exchange adds to π exchange for the central part, but reduces the tensor part of the interaction.

The nuclear medium effect is shown diagramatically in Figure IV.3 .

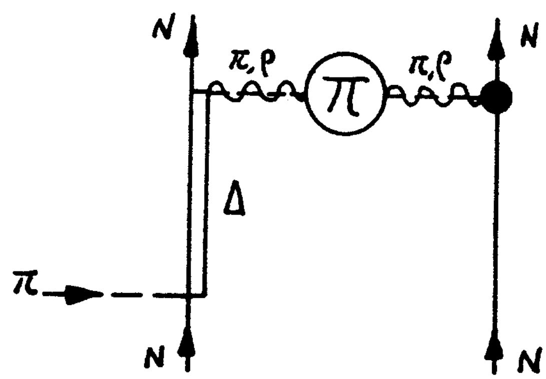

Figure IV.3 The direct channel of the P wave pion rescattering absorption mechanism with intermediate $\pi + \rho$ exchange and the medium effects

The calculation is similar to that of the S wave. That is, the calculation is divided into two categories. The first is P wave absorption in nuclear matter [Ref. IV.2,4] . The Fermi gas model is used to construct the initial paired nucleon wave function and plane waves are used to describe the final knockout nucleons. In the second category we deal with P wave pion aborption in a finite nucleus which is discussed later in this section. The results for the absorption parameter Im C_0 for nuclear matter, using various form factors, with and without medium corrections are shown in Figure IV.4.

Details of numerical analysis reveals that the transition of paired nucleons coupled to $L = 0$ to $L' = 2$ channel, via the tensor interaction dominates. So, for the sake of simplicity only the contribution of this channel is shown in Figure IV.4 .

Monopole form factors i.e. Equation (2-4-5) with standard cut off mass parameters i.e. $\Lambda_{\pi} = 1200$ Mev and $\Lambda_{\rho} = 2000$ Mev were used. These were obtained by fitting to experiments on the absorption of pions by deuterons. They correspond to a nucleon source size of 0.25 fm for ρ's and of 0.4 fm for π's .

For threshold pions Im C_0 is $0.043\,\mu^{-6}$. The value of Im C_0 estimated from pionic atom level width varies between $0.036\,\mu^{-6}$ to $0.07\,\mu^{-6}$ [Ref. IV. 5-7] depending on the LLEE parameter.

By confining our calculations to the standard cut off mass parameter i.e. parameters used in $\pi d \rightarrow pp$ [Ref. IV.8] and by comparing curves–·–·–, π, exchange only, and – – –, $\pi + \rho$ exchange, of Figure IV.4 one can see that the destructive interference between π and ρ exchange in the tensor part of the interaction is essential in obtaining a reasonable value of Im C_0. It is interesting to note that curve – – – Figure IV.4, that is combined π and ρ meson exchange with the standard cut off mass parameters, can almost be reproduced by π ex-

Figure IV.4

The imaginary component of the two-body P wave pion absorption parameter of nuclear matter vs. pion momentum.
Dashed-dot line corresponds to one π exchange with Λ_π = 1200 Mev and no medium effects.
Dashed line corresponds to $\pi + \rho$ exchange with Λ_π= 1200 Mev and Λ_ρ= 2000 Mev and no medium effects.
Dashed-dot-dot line corresponds to one π exchange with Λ_π = 1200 Mev and medium effects at λ = 1.5 .
Solid line corresponds to $\pi + \rho$ exchange with Λ_π = 1200 Mev and Λ_ρ = 2000 Mev and medium effects at λ = 1.5 .
Dashed-slash line corresponds to one π exchange with Λ_π = 700 Mev and no medium effects.

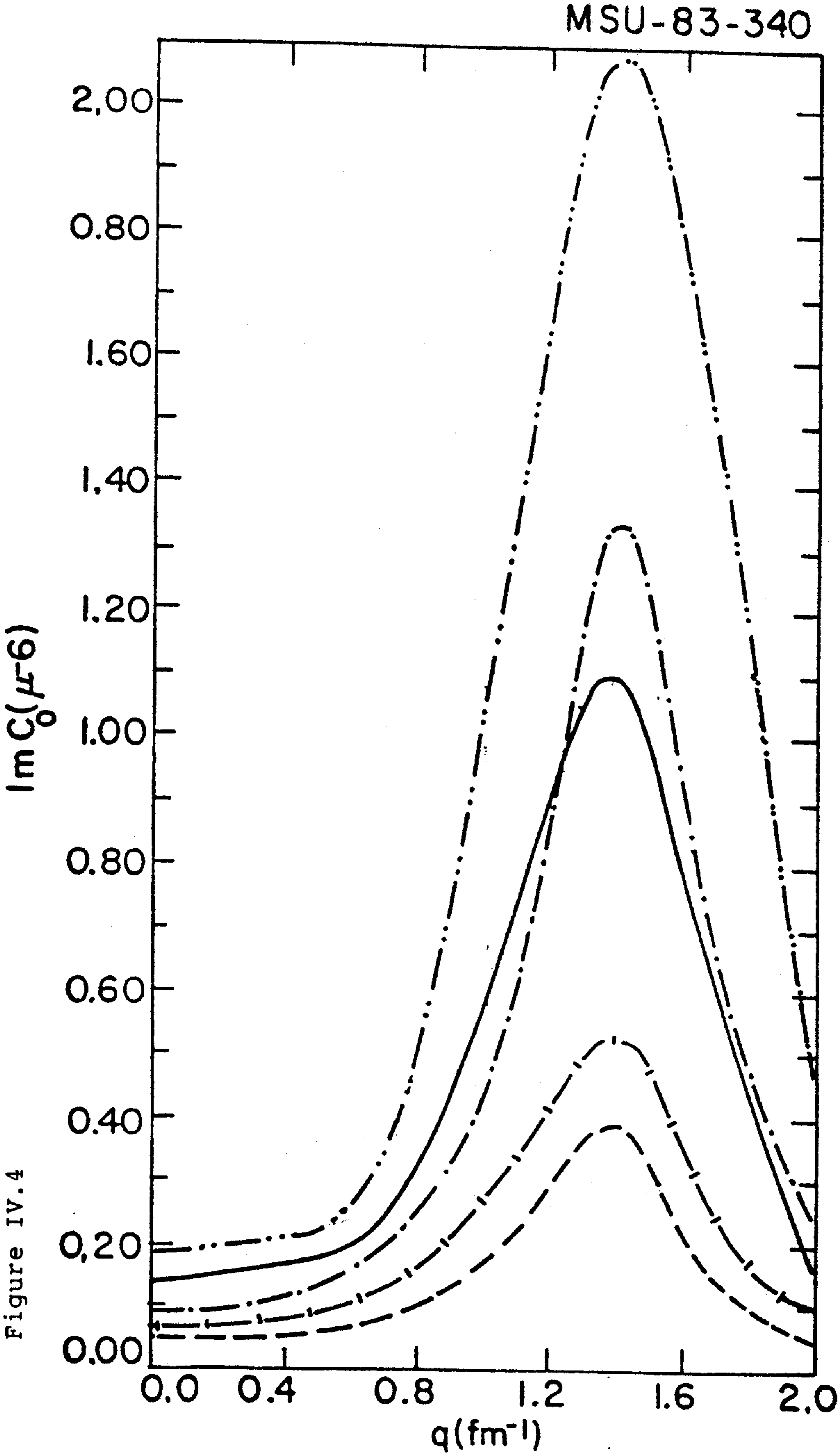
MSU-83-340
2.00
0.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
$\mathrm{Im}\,C_0(\mu^{-6})$
0.0
0.4
0.8
1.2
1.6
2.0
q(fm^{-1})

Figure IV.4

change alone with a reduced cut off parameter, Λ_π = 700 Mev, curve —|—|—|– Figure IV.4 . This corresponds to a nucleon source size of 0.7 fm for π's , which is closer to the physical size of the nucleon, and that given by the Cloudy Bag Model of Thomas et al, [Ref. IV.9] .

To include nuclear matter effects the central and tensor components of the OBE potential i.e. Equation (4-1-8a,b), are replaced by the following integrals

$$Y_n(\mu^* r) \longrightarrow \frac{2}{\pi \mu^{*n+1}} \int_0^\infty \frac{\Gamma(k) f_\pi^2(k)\, k^{n+2} j_n(kr)}{\mu^2 + k^2 - \omega'^2 + \Pi(k)}\, dk \qquad (4\text{-}1\text{-}9)$$

$n = 0, 2$

where the vertex renormalization $\Gamma(k)$ and pion's self-energy $\Pi(k)$ are given by Equations (3-1-10) and (2-4-2) respectively.

By comparing curve----, standard parameters with no medium effect, and—,standard parameters with medium effect for λ = 1.5, we conclude that the many-body correction increases Im C_0 at all energies. The P wave pion absorption calculation for ^{4}He proceeds as in the S wave case.

The result is shown in Figure IV.5 . The general feature of Im C_0 vs. pion momentum is similar to the behavior of Im C_0 in nuclear matter. The dashed curve shows the result, without medium correction, for the standard

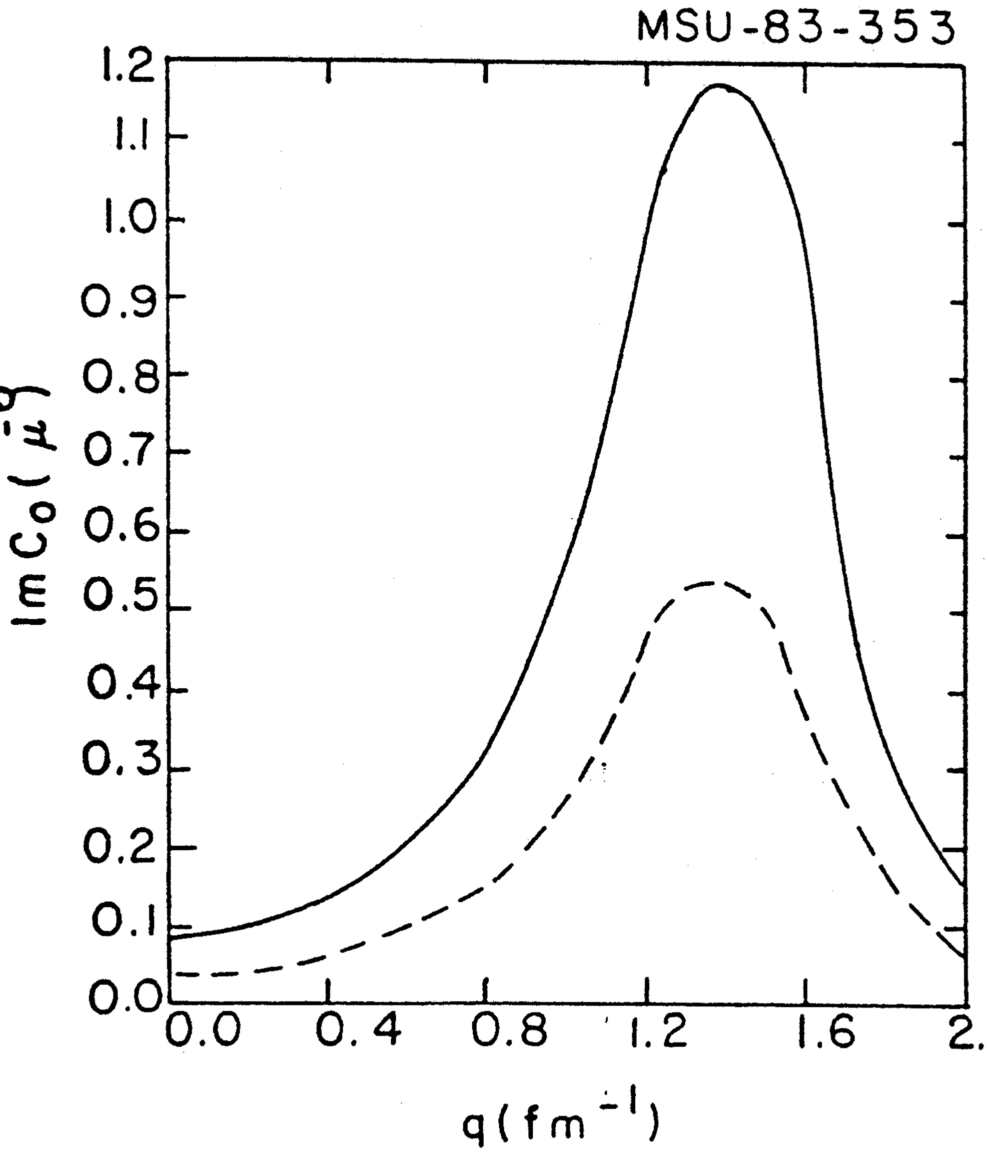

Figure IV.5

The imaginary component of the two-body P wave pion absorption parameter of ^{4}He vs. pion momentum.
Dashed line corresponds to $\pi + \rho$ exchange with Λ_π = 1200 Mev and Λ_ρ = 2000 Mev and no medium effects.
Solid line corresponds to $\pi + \rho$ exchange with Λ_π = 1200 Mev and Λ_ρ = 2000 Mev with medium effects at $\lambda = 1.5$.

parameters. The effect of including the medium corrections for the same parameters is shown by the solid line. The overall amplification due to the many-body effect is quite similar to the amplification in the nuclear matter case. So one concludes that, contrary to the case of S wave pions, the medium correction does have an appreciable effect even in a small finite nucleus, for P wave absorption.

To conclude we have calculated S and P wave rescattering absorption and the second order S wave repulsive contribution to the optical potentials using parameters fitted to pion absorption on the deuteron. These give, in general, too small an effect. However, values are amplified when we take into account an effect not present in the deuteron, the self energy of a propagating pion in the average nuclear field. In fact the pion interacts so strongly with the nuclear medium that this amplification could become extremely large (nuclear opalescence , pion condensation) but for the damping casued by two nucleon correlations and other effects parametrized by the LLEE variable λ or Landau parameter g' .
For $g' = 0.4$, $\lambda = 1.2$ nuclear opalescent effects would be prominent. However g' is usually estimated to be $g' = 0.65 - 0.85$ with $\lambda = 1.9 - 2.5$. We used $\lambda = 1.5$ so we probably have a slight overestimate of medium corrections. The results are the same for P wave absorption, whether we use the conventional $\pi + \rho$ exchange with nu-

cleon source size $\cong$ 0.4 fm, or π exchange alone with source size $\cong$ 0.7 fm . However the S wave effects, which depends on pion exchange alone would become negligibly small in the latter case.

4-2 Isospin Dependence of Pion Absorption by Nucleon Pairs in the He Isotopes

The P wave rescattering mechanism also predicts the ratio of the absorption of a pion on a pair of nucleons in a T = 0 state to that on a pair in the T = 1 state, which is the only channel for the process $\pi^- + (pp) \rightarrow pn$.

This ratio $R = \sigma(T=0)/\sigma(T=1)$ was measured recently by Ashery et al [Ref. IV.10] in experiments with 165 Mev π^+ and π^- on ^3He and ^4He . They found that $R \cong 50$, with large errors, i.e. absorption on two nucleons in a T = 1 state is extremely improbable.

The absorption amplitude is the sum of two terms, one from the direct, the other from the crossed channel, shown in Figure IV.1 .

The amplitudes are

$$T_a = f_{\pi NN}/\mu \, f_{\pi N\Delta}/\mu \; \vec{\sigma}_1 \cdot \vec{k}_1 \, \vec{\tau}\,' \cdot D_\pi(k_1) \, \vec{S}_2^{\dagger} \cdot \vec{k}_1 \cdot \vec{T}_2^{\dagger} \, G_D(P_\Delta) \quad (4\text{-}2\text{-}1a)$$
$$\vec{S}_2 \cdot \vec{k} \, \vec{T}_{2\lambda} + 1 \leftrightarrows 2$$

$$T_b = f_{\pi NN}/\mu \, f_{\pi N\Delta}/\mu \; \vec{\sigma}_1 \cdot \vec{k}_1 \, \vec{\tau}\,' \cdot D_\pi(k_1) \, \vec{S}_2 \cdot \vec{k}_1 \, \vec{T}_2 \, G_c(P_\Delta)$$
$$\vec{S}_2^{\dagger} \cdot \vec{k} \, \vec{T}_{2\lambda} + 1 \leftrightarrows 2 \quad (4\text{-}2\text{-}1b)$$

where the coupling constants $f_{\pi NN}$ and $f_{\pi N\Delta}$ are given in the previous sections. The $\vec{\sigma}$ and $\vec{\tau}$ are the usual Pauli's nucleon spin and isospin operators, the $\vec{S}$ and $\vec{T}$ are the transition spin and isospin operators of Δ isobar [Ref. IV.11] .

G_D and G_c are the direct and crossed channel isobar propagators,

$$G_D = m_\Delta - m + P_\Delta^2/2m_\Delta - \omega - \frac{i}{2}\Gamma_\Delta \qquad (4\text{-}2\text{-}2a)$$

$$G_C = m_\Delta - m + P_\Delta^2/2m_\Delta + \omega \qquad (4\text{-}2\text{-}2b)$$

If we take the pion orbital angular momentum relative to the nucleon pair to be $\ell_\pi = 1$, we can easily see that the direct channel gives no contribution to absorption by a T = 1 nucleon pair in a state of relative angular momentum L = 0 , and hence by the Pauli principle S = 0 . The intermediate state Δ, n , then has spin parity $J^\pi = 1^+$. It could have isospin T = 1 or 2 but as it has to decay into two nucleons, only T = 1 is allowed. So the final state NN has to have $J^\pi = 1^+$, T = 1 , i.e. S = 1 . L = 0 or 2 and T = 1 . But such a state is forbidden by the Pauli principle. So the only contribution comes from the crossed channel. Similarly the only contribution to absorption from a T = 0 initial pair state comes from the direct channel.

Then for $\ell_\pi = 1$, $\frac{d\sigma(T=0)}{d\sigma(T=1)} = \left|\frac{q_D}{q_c}\right|^2$ as first pointed out by Chai and Riska [Ref. IV.4] when discussing the ratio of final nn to np pairs produced in the capture of pions from the 2p level in pionic atoms. This gives a ratio about = 200 for 165 Mev pions, and is independent of any mechanism except the assumption that absorption is a two step process, proceeding through the formation of a Δ.

It remains to be seen whether other pion partial waves, $\ell_\pi = 0$ and 2 contribute to the small $d\sigma(T = 0)$ cross section. To simplify the calculation, we used only pion exchange with a cut off parameter $\Lambda_\pi = 0.8$ Gev instead of $\pi + \rho$ exchange with $\Lambda_\pi = 1.2$ and $\Lambda_\rho = 2.0$ Gev as in the standard model, as we have already seen that this gives equivalent results in the calculation of the absorption rate. The results are given in Table IV.1 .

Initial	l_π	T'	S'	L'	σ				
					$k[\mu]$ = 0.5	1.0	1.5	2.0	2.5
$T = 0, S = 1, L = 0$	0	1	1	1	0.027	0.40	2.9	9.7	7.3
	1	1	0	2	1*	4.9	21	52	30
	2	1	1	3	0.006	0.11	1.0	4.2	3.6
$T = 1, S = 0, L = 0$	0	1	1	1	0.008	0.086	0.44	1.1	0.63
	1	0	1	2	0.076	0.17	0.24	0.26	0.26
	2	1	1	3	0.002	0.023	0.14	0.40	0.26

TABLE IV.1

Pion absorption cross section in several channels as a **function** of pion momentum k are compared, where l is the **angular** momentum of the incoming pion. These numbers are **normalized** to the (T,S,L) = (0,1,0) to (T',S',L') = (1,0,2) **transition** at k = 0.5 as indicated by * .

This shows that $l_\pi = 1$ is indeed dominant in the T = 0 channel, but that the other partial waves are more important, except at threshold, in the T = 1 channel. Adding up the contributions from all three partial waves we find the ratio is reduced to R = 40 at 165 Mev consistant with the experimental results [Ref. IV.13], as plotted in Figure IV.6.

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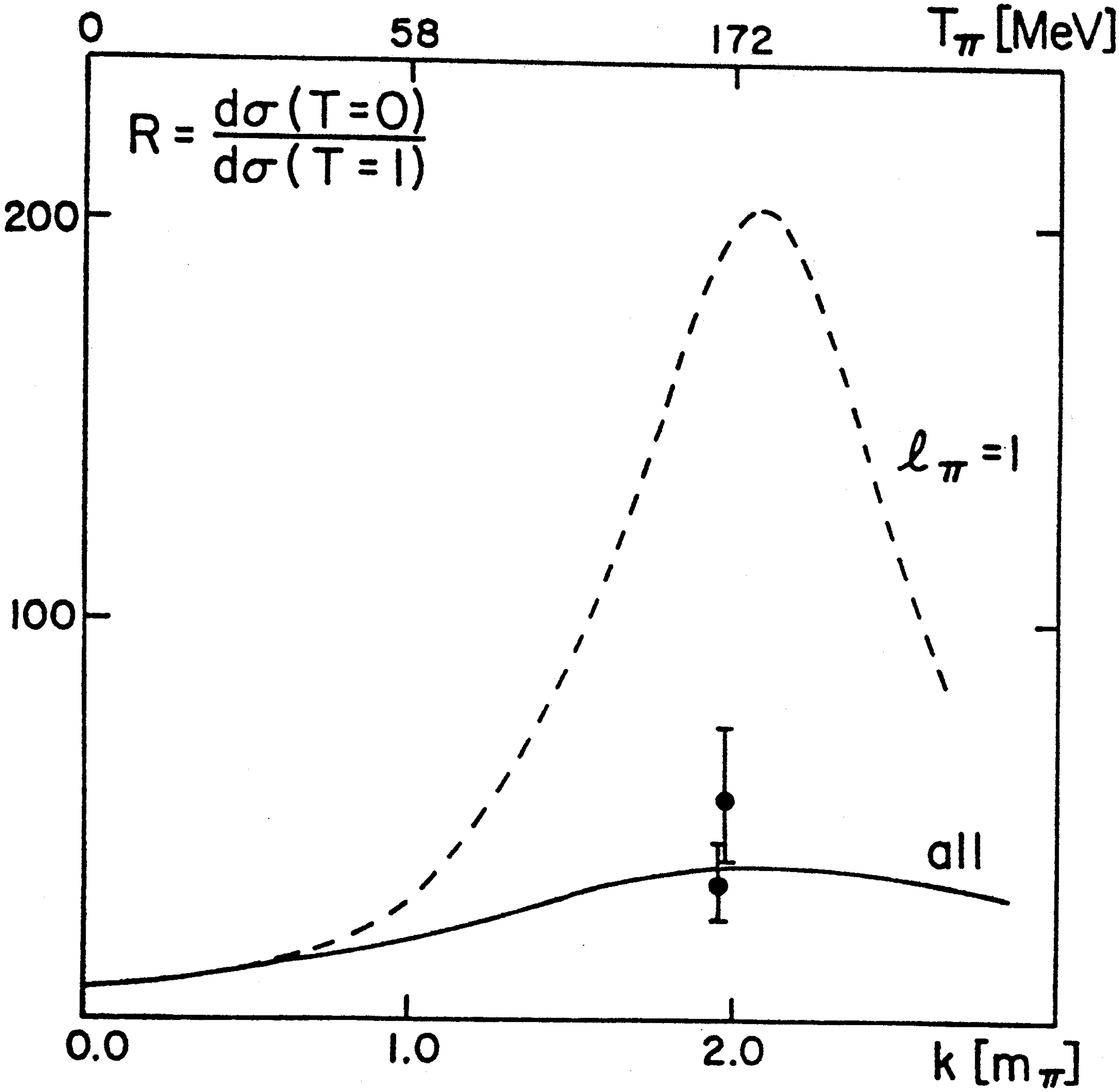

Figure IV.6

The pion absorption ratio $R = \frac{d\sigma(T=0)}{d\sigma(T=1)}$ vs. pion momentum.
Dashed line corresponds to $\ell_\pi = 1$ and π exchange alone with Λ_π = 800 Mev.
Solid line corresponds to all of the possible partial wave contributions.

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Chapter V

AN ALTERNATIVE TREATMENT OF P WAVE PION ABSORPTION IN THE OPTICAL MODEL

5-1 We would like to mention another way of incorporating P wave pion absorption in the optical model, i.e. the Δ-h model. previously we parameterized the absorption as a term in the optical model

$$2\omega U_{abs} = -4\pi \vec{\nabla} C_0 \rho^2 \vec{\nabla} \tag{5-1-1}$$

where C_0 was calculated via the absorption rate, Equation (3-2-4) . This gave a resonant term in the optical potential adding to the resonant behavior without absorption. Both terms are due to the formation of the intermediate Δ resonance, so it may be better to display this explicitly (Δ-h model) . The P wave part of the optical potential is written without absorption,

$$2\omega U_{opt} = \vec{\nabla}\, 4\pi c_0 \rho\, \vec{\nabla} \tag{5-1-2}$$

where

$$4\pi c_0 \rho = \frac{4}{9} f^{*2}/\mu^2 \left[U_\Delta(\omega) + U_\Delta(-\omega) \right] \tag{5-1-3}$$

and

$$U_\Delta(\omega) = \rho(r) / \omega_R - \omega - \frac{i}{2}\Gamma_{free} \qquad (5\text{-}1\text{-}4)$$

Here Γ_{free} is the isobar width given by Equation (2-4-11) due to the process $\Delta \rightarrow \pi + N$.

In the presence of nuclear matter Γ_{free} is modified
1) by Pauli blocking in the nuclear medium $\Gamma_{free} \rightarrow Q\Gamma_{free}$
2) by true absorption $\Delta +$ N $\rightarrow$ N + N . This second process depends on the nuclear density. Its effect is to increase the width by an additional width, the absorption width $\Gamma_a = \Gamma_1 \rho(r)/\rho_0$ so in this formalism the effect of the nuclear medium is to change Γ_{free} , $\Gamma_{free} \rightarrow Q\Gamma_{free} + \Gamma_1 \rho(r)/\rho_0$. So $U_\Delta(\omega) \rightarrow \tilde{U}_\Delta(\omega)$, where

$$\tilde{U}_\Delta(\omega) = \rho(r) / \omega_R - \omega - \frac{i}{2}\left(Q\Gamma_{free} + \Gamma_1 \rho(r)/\rho_0\right) \qquad (5\text{-}1\text{-}5)$$

and

$$2\omega U_{opt} = \frac{4}{9} f^{*2}/\mu^2 \, \vec{\nabla}\left[\tilde{U}_\Delta(\omega) + \tilde{U}_\Delta(-\omega)\right]\vec{\nabla} \qquad (5\text{-}1\text{-}6)$$

The basic task is then to calculate Γ_1 i.e. the rate of the process $\Delta +$ N $\rightarrow$ N + N .

As we have already seen, the dominant process in absorption goes from, for π^+, a (Δ^{++} n) intermediate state with quantum numbers T = 1 L = 0 S = 2 which goes via the tensor force to a final state of two protons with quantum numbers T'= 1 L'= 2 S'= 0, so we consider only this process. The calculation is done for ^{4}He replacing a proton by a Δ^{++} isobar with the same spatial wave function, and the standard " $\pi + \rho$ " model is used for the process.

The transition rate is

$$\Gamma = 2\pi \sum_{fi} \delta(E_f - E_i - \omega) \, |T_{fi}|^2 \tag{5-1-7}$$

For pion exchange the vertices are given by

$$\mathcal{L}_{\pi NN} = f/\mu \; \bar{\psi} \gamma_5 \gamma_\nu \partial_\nu \; \vec{\tau} . \vec{\phi} \; \psi \tag{5-1-8}$$

$$\mathcal{L}_{\pi N\Delta} = f^*/\mu \; \bar{\psi} \; \partial_\nu \vec{T} . \vec{\phi} \; \psi_\nu + h.c. \tag{5-1-9}$$

where ψ , ϕ , ψ_ν are the nucleon, pion and isobar's fields and τ and T are the nucleon isospin and isobar's transition isospin operator [Ref. V.1] , and coupling constants are $f^2/4\pi$ = 0.08 and $f^{*2}/4\pi$ = 0.32 .

The transition operator is then

$$T(N\Delta \to NN) = f f^*/\mu^2 \, \mu^{*3}/12\pi \left[\vec{S}_1^\dagger . \vec{\sigma}_2 \, Y_0(x) + S_{12} Y_2(x) \right] \vec{T}_1^\dagger . \vec{\tau}_2 \tag{5-1-10}$$

where $\mu^* = \sqrt{\mu^2 - \omega'^2}$ is the pion effective mass and ω' = 1/2 μ is the resacttered pion energy, $\vec{\sigma}$ is nucleon spin and $\vec{S}$ is the isobar transition spin operator [Ref.V.1].

Only the second term is important

$$Y_2(x) = (1 + 3/x + 3/x^2)\, Y_0(x) \tag{5-1-11}$$

$$S_{12} = 3\vec{S}_1^{\dagger}\cdot\hat{r}\,\vec{\sigma}_2\cdot\hat{r} - \vec{S}_1^{\dagger}\cdot\vec{\sigma}_2 \tag{5-1-12}$$

The matrix elements of the isospin operator give

$$\langle T'(\tfrac{1}{2}\tfrac{1}{2})M_{T'} |\, \vec{T}_1^{\dagger}\cdot\vec{\tau}_2 \,| T(\tfrac{3}{2}\tfrac{1}{2})M_T\rangle = (-)^T \delta_{TT'}\delta_{M_T M_{T'}} \begin{Bmatrix} 3/2 & 1/2 & T \\ 1/2 & 1/2 & T' \end{Bmatrix} \langle \tfrac{1}{2} \| \vec{T}_1^{\dagger} \| \tfrac{3}{2}\rangle \langle \tfrac{1}{2} \| \tau_2 \| \tfrac{1}{2}\rangle \tag{5-1-13}$$

while for the spin operator

$$\langle s'(\tfrac{1}{2}\tfrac{1}{2})0 |\, S_{12}^{\dagger} \,| s(\tfrac{3}{2}\tfrac{1}{2})2\rangle = \sqrt{\frac{24\pi}{5}}\; Y_{2M}^{*}(\hat{r}) \tag{5-1-14}$$

where M is the z component of the initial spin state.

For ^4He the harmonic oscillator basis wave functions are $\exp(-\frac{1}{2}\nu r_1^2)$, $\exp(-\frac{1}{2}\nu r_2^2)$, for each " nucleon " so that initial pair wave function hehaves as $\exp(-\nu R^2)\exp(-\frac{1}{4}\nu r^2)$, and the final pair wave function becomes

$\psi_f(1,2) = \frac{2}{\Omega\sqrt{2}} \exp(i\vec{k}\cdot\vec{R})\cos(\vec{k}_f\cdot\vec{r})$ here CM and relative co-ordinates and momenta are defined as

$$\vec{R} = \tfrac{1}{2}(\vec{r}_1 + \vec{r}_2) \qquad \vec{k} = \vec{k}_1 + \vec{k}_2$$
$$\& \tag{5-1-15}$$
$$\vec{r} = \vec{r}_1 - \vec{r}_2 \qquad \vec{k}_f = \tfrac{1}{2}(\vec{k}_1 - \vec{k}_2)$$

To handle the final state relative wave functions, expansion (3-3-11) is used of which only the $l = 2$ term gives a contribution.

To calculate the rate, the δ - function giving conservation of energy is

$$\sum_f \delta(E_f - E_i - \omega) \to \frac{\Omega}{(2\pi)^3} \int \delta(k_f^2/2m + B - \mu) k_f^2 dk_f \, d\Omega_{\hat{k}_f} \qquad (5\text{-}1\text{-}16)$$

where $E_i = -B$, B is the nucleon binding energy and m is the nucleon mass. The integration yields

$$\Gamma_1 = 3781 \, |I_{02}|^2 \qquad (5\text{-}1\text{-}17)$$

where I_{02} is given by

$$I_{02} = \int_{r_c}^{\infty} J_2(k_f r) Y_2(\mu^* r) \exp(-\tfrac{1}{4} v r^2) r^2 dr \qquad (5\text{-}1\text{-}18)$$

where $K_f = \sqrt{m(\mu - B)}$, and r_c is a cut off in the radial integration to take account of short-range correlations.

We now put in form factors at the vertices. To simplify matter we take half-dipole form factors

$$f(q^2) = (\tilde{\lambda}^2 - \mu^2/\tilde{\lambda}^2 + q^2)^{1/2} \qquad (5\text{-}1\text{-}19)$$

at each interaction vertex instead of the regular dipole form factors, but keeping $< r^2 >$ the same, i.e. $\tilde{\Lambda} = 1/\sqrt{2}\Lambda$.

These form factors modify the radial integral

$$\mu^{*3}\int_{r_c}^{\infty} J_2(k_f r)\, Y_2(\mu^{*}r)\exp(-\tfrac{1}{4}vr^2)\, r^2 dr \rightarrow \qquad (5\text{-}1\text{-}20)$$

$$\mu^{*3}\int_{r_c}^{\infty} J_2(k_f r)\, Y_2(\mu^{*}r)\exp(-\tfrac{1}{4}vr^2)\, r^2 dr - \tilde{\Lambda}^3\int_{r_c}^{\infty} J_2(k_f r)\, Y_2(\tilde{\Lambda}r)\exp(-\tfrac{1}{4}vr^2)\, r^2 dr$$

and also Γ_1 gains a multiplicative factor $\left(\tilde{\Lambda}^2-\mu^2/\tilde{\Lambda}^2-\mu^{*2}\right)^2$.

ρ - exchange is taken into account in the same way as in Section (4-1) .

Figure V.1 shows the behavior of Γ_1 vs. cut off radius r_c for the standard parameter set. Γ_1 is relatively insensitive to this parameter up to $r_c \cong 1$ fm.

We conclude that short range correlations are not crucial, with the standard form factors, to the evaluation of Γ_1 . The value of $\Gamma_1 \cong 55$ Mev . This is in order of magnitude agreement with the empirical value of $\Gamma_1 \leq 100$ Mev.

Medium corrections would presumably increase these values, in better agreement with empirical values, but we have not done this calculation.

It should be mentioned that to get agreement with experiment using the Δ - h model, a more complex non-local expresion has to be used, where the single particle potential seen by the Δ in nuclear matter, including a strong spin-orbit interaction, is important.

A comparision of the forms of this simplified Δ - h optical potential labelled " microscopic " with the Kisslinger type labelled " macroscopic " , is shown in Table V.1 .

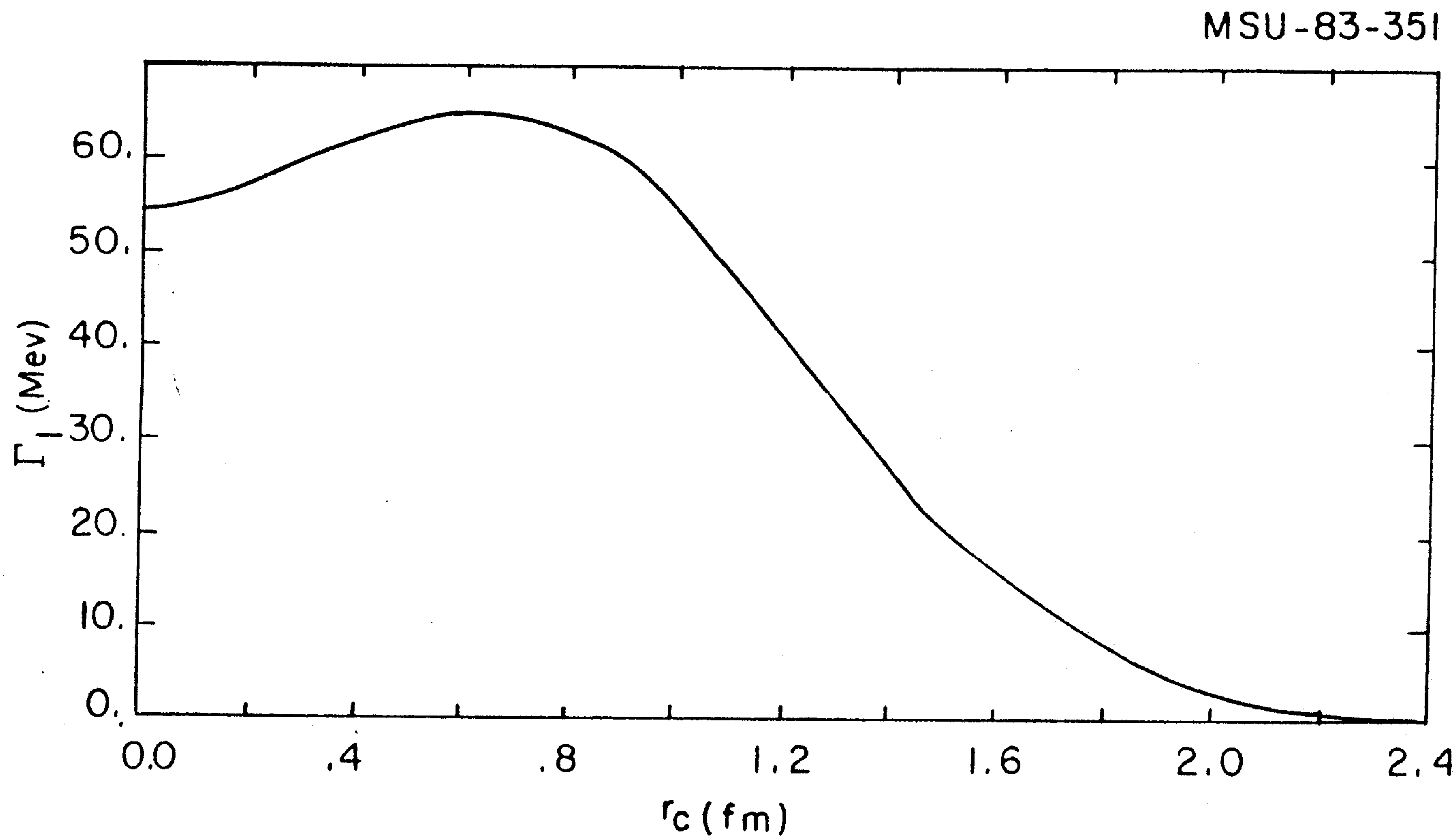


Figure V.1

The rate of the $\Delta + N \rightarrow N + N$ process of ^4He at threshold vs. the cut off radius.

TABLE V.1 A comparison of the forms of Δ-h optical opential labelled " Microscopic " with the Kisslinger type labelled " Macroscopic "

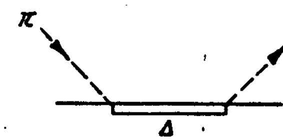
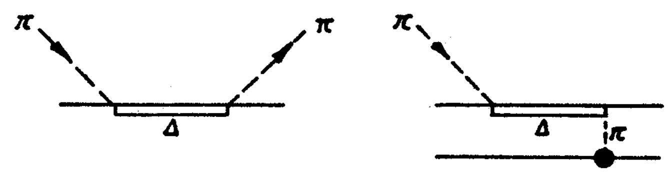
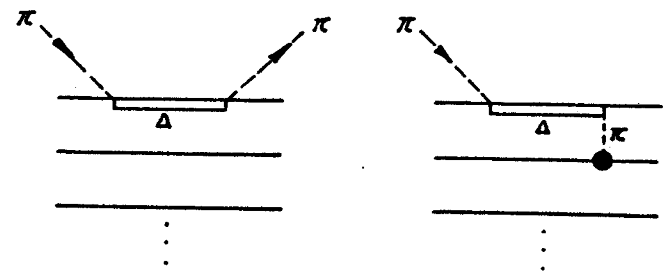
Macroscopic	Microscopic
1. P wave pion-nucleon scattering parameter c in terms of phase shift	The same quantity is calculated microscopically based on the following diagram 
$$C_0 = \frac{4}{3K_{C.M.}k^2} \sin\delta_{33} \exp(i\delta_{33})$$	$$C_0 = \frac{4}{9}(f_\Delta^2/4\pi)\frac{1}{\mu^2}\left(\frac{K_{CM}}{K}\right)^2 \frac{\omega_\Delta}{\omega} \frac{1}{\omega_\Delta - \omega - \frac{i}{2}\Gamma}$$
2. P wave optical potential at zeroth order	P wave optical potential at zeroth order
$$\frac{2\omega}{\rho\vec{k}\cdot\vec{k}'} U_{opt} = 4\pi c_0$$	$$\frac{2\omega}{\rho\hat{\vec{k}}\cdot\hat{\vec{k}}'} U_{opt} = 4\pi c_0$$
3. Parameterization of the optical potential in terms of one and two-boby mechanism	Parameterization of the optical potential in terms of one and two-body mechanism 
$$\frac{2\omega}{\rho\vec{k}\cdot\vec{k}'} U_{opt} = 4\pi(C_0 + C_0\rho)$$	$$\frac{2\omega}{\rho\vec{k}\cdot\vec{k}} U_{opt} = \frac{4}{9}(f_\Delta^2/4\pi)\frac{1}{\mu^2}\left(\frac{K_{CM}}{K}\right)^2 \frac{\omega_\Delta}{\omega} \frac{1}{\omega_\Delta - \omega - \frac{i}{2}(\Gamma + \xi(\rho/\rho_0))}$$

TABLE V.1 Continued.

4. Modification of the third step due to the many body problem

$$\frac{2\omega}{\rho\vec{k}\cdot\vec{k}'}U_{opt} = f = 4\pi\left(\frac{c_0 + C_0\rho}{1+\frac{4}{3}\pi\lambda(c_0\rho + C_0\rho^2)}\right)$$

Modification of the thired step due to the many body problem



$$\frac{2\omega}{\rho\vec{k}\cdot\vec{k}'}U_{opt} = g = \frac{4}{9}(f_\Delta^2/4\pi)\frac{1}{\mu^2}\left(\frac{k_{CM}}{k}\right)^2\frac{\omega_\Delta}{\omega}\frac{1}{\omega_\Delta-\omega-\frac{i}{2}(\Gamma Q+\Gamma_1\rho/\rho_0)}$$

A plot of the Real and Imaginary parts of these potentials f,g at $\rho = 1/2\, \rho_o$, as a function of energy, together with that given by the impulse approximation, c_o , is shown in Figures V.2,3 . The principal effect of changes from the impulse approximation is to increase the imaginary part at threshold, and to push the effective resonance slightly upwards in energy, in qualitative agreement with empirical results.

lastly we consider briefly and qualitatively the region of validity of the model taking into account the fact that nucleons and Δ 's are extended objects, roughly " bags " containing three quarks. If there was general agreement about two nucleon dynamics at short distances, this would be simple, but there are at least three competing versions of the bag model.

1) the strong coupling model of G.E. Brown and collaborators [Ref. V.2] . This has a nucleon bag radius of 0.4 - 0.5 fm, with a strongly coupled pion cloud around it. This model preserves conventional nuclear physics with meson exchange including the standard " $\pi + \rho$ " model. Baryons are " hard " and kept apart by strong forces. Advantages claimed are that six-quark bags of overlapping nucleons are very massive, giving strong repulsion at short distances, and that the excitation spectrum of baryons is given correctly. 2) The second model, the cloudy bag of A.W. Thomas and collaborators [Ref. V.3] , has a nucleon bag radius of 0.7 - 0.8 fm, with a relatively weakly coupled pion field.

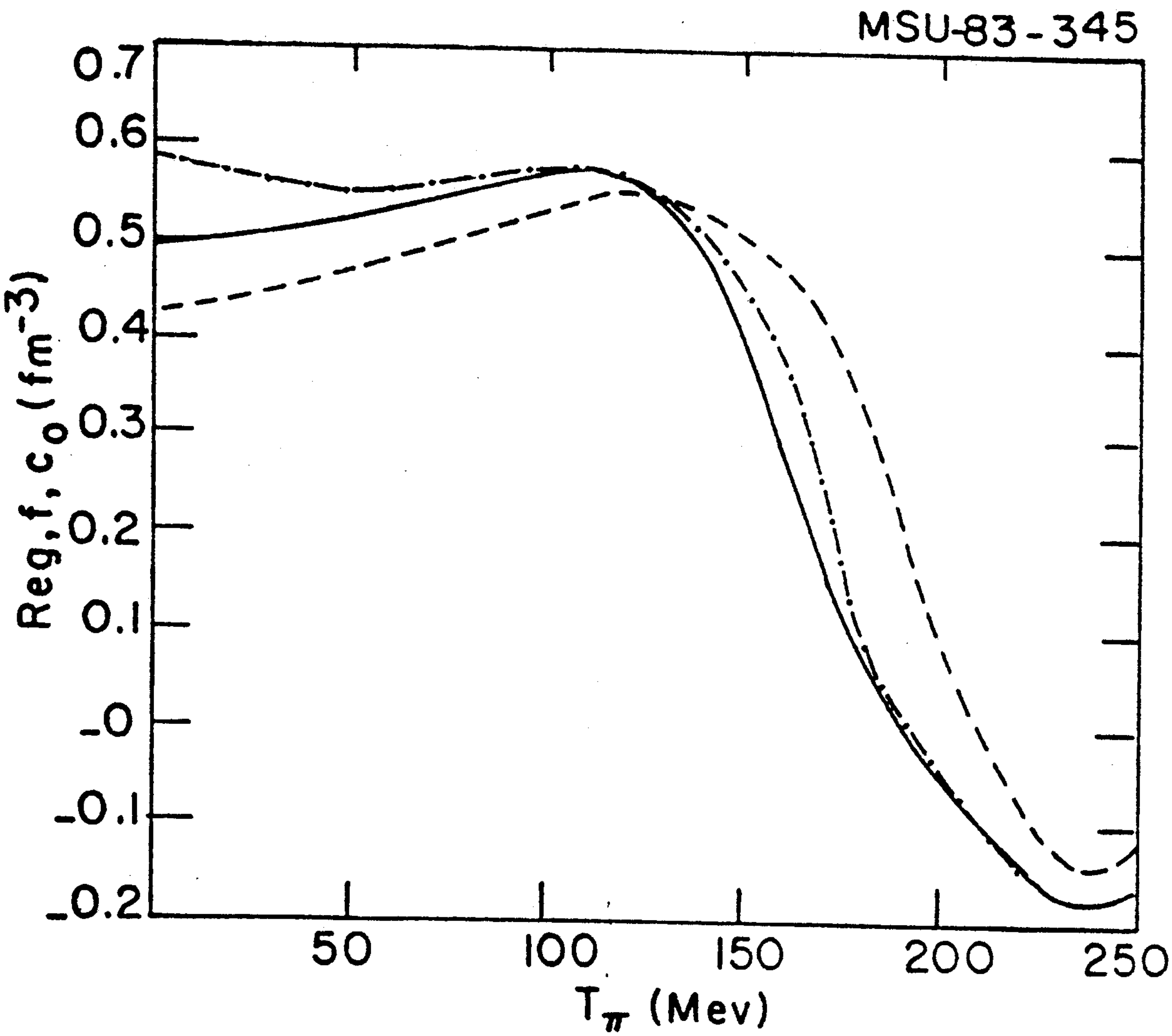


Figure V.2

The real component of the P wave pion-nucleus optical potential vs. pion kinetic energy.
Dashed-dot line corresponds to the microscopic treatment (g) with ρ = 0.5 nuclear matter density.
Solid line corresponds to the impulse approximation (c_o)
Dashed line corresponds to the macroscopic treatment (f) with ρ = 0.5 nuclear matter density and λ = 1.5.

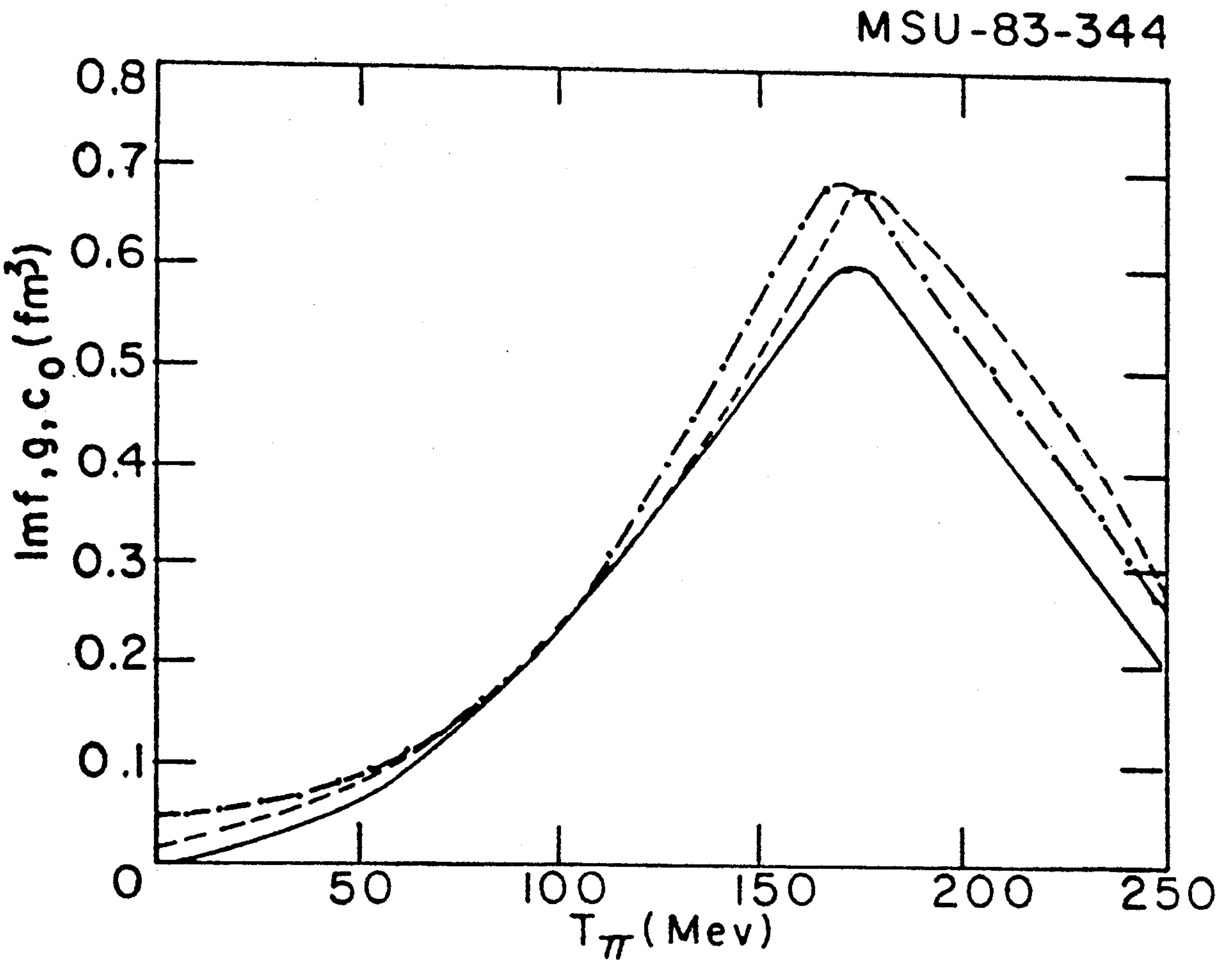


Figure V.3

The imaginary component of the P wave pion-nucleus optical potential vs. pion kinetic energy.
Dashed-dot line corresponds to the microscopic treatment (g) with ρ = 0.5 nuclear matter density.
Solid line corresponds to the impulse approximation (c_o)
Dashed line corresponds to the macroscopic treatment (f) with ρ = 0.5 nuclear matter density and λ = 1.5

3) The original M.I.T. bag [Ref. V.4] has a radius of ≃ 1 fm and makes the nucleon very " soft " . Nucleons interpenetrate easily, but remain in quark clusters of three. Inside a nucleus these clusters of three quarks behave like a nucleon of mass 0.75 m which, it is claimed, is seen in e e' p experiments in nuclei. A disadvantage is that six quark bags are not very massive, so that the repulsive force seen in N-N scattering experiments must be due entirely to the large, ≃ 2 fm, non locality in the interaction arising from quark dynamics. The point is that if nucleons are bags of radius R then, when their centers are a distance r = 2R apart, the bags begin to touch and quark-gluon dynamics rather than meson exchange begins to be the predominant interaction. Actually the percentage overlap increasese only slowly with decreasing distance between centers. At r = 1.2 R the percentage overlap is ≃ 12% . At this point we might expect the bag to collapse into a six-quark bag, as the quarks are relativistic and can rearrange the distribution instantaneously. We could take this to be the upper limit for a critical distance, r_c below which the meson exchange model would be invalid. The lower limit for r_c we could take empirically from nucleon-nucleon scattering as the hard core radius ≃ 0.5 fm.

The form factors used in our calculation would correspond to nucleon size if the root mean square radius given by the parameter $\Lambda \cong$ bag radius R.

The meson exchange model might be expected to give the dominant contribution to absorption if most of the contribution comes from that part of the radial integrals from distances $r \gtrsim r_c$. Ignorring this for the moment, Figure V.4 shows the P wave absorption parameter Im C_0 as a function of pion momentum for 4-cases, pion exchange only with form factor parameter Λ = 800 , 700 and 400 Mev, corresponding to root mean square radii of 0.6 , 0.7 and 1.2 fm respectively, and the standard " $\pi + \rho$ " model with Λ_π= 1200 Mev, or root mean square radius of 0.4 fm. It is seen that π exchange only with Λ = 700 Mev, or radius of o.7 fm, agrees with the standard model. However π - exchange alone with Λ = 400 Mev, i.e. radius 1.2 fm gives much too small a result. So if the M.I.T. bag radius is correct, nothing comes from meson exchange and everything is due to quark-gluon dynamics. However, Λ = 700 Mev with radius 0.7 fm, corresponding to the cloudy bag model is O.K. The stsndard model is consistant, naturally, with G.E. Brown's little bag model.

Now let us look at the effect of a lower cut-off due to overlapping sources. Figure V.5 shows Im C_0 at threshold as a function of the lower cut off radius r_c in the radial integral. For the case of interest, pion exchange alone with Λ_π = 700 Mev or 800 Mev, and the standard model, the results are quite insensitive to a cut off radius up to $r_c \cong$ 0.9 fm, and so consistant with the cloudy bag model, but again not with the M.I.T. bag radius.

We conclude that as far as the P wave absorption is

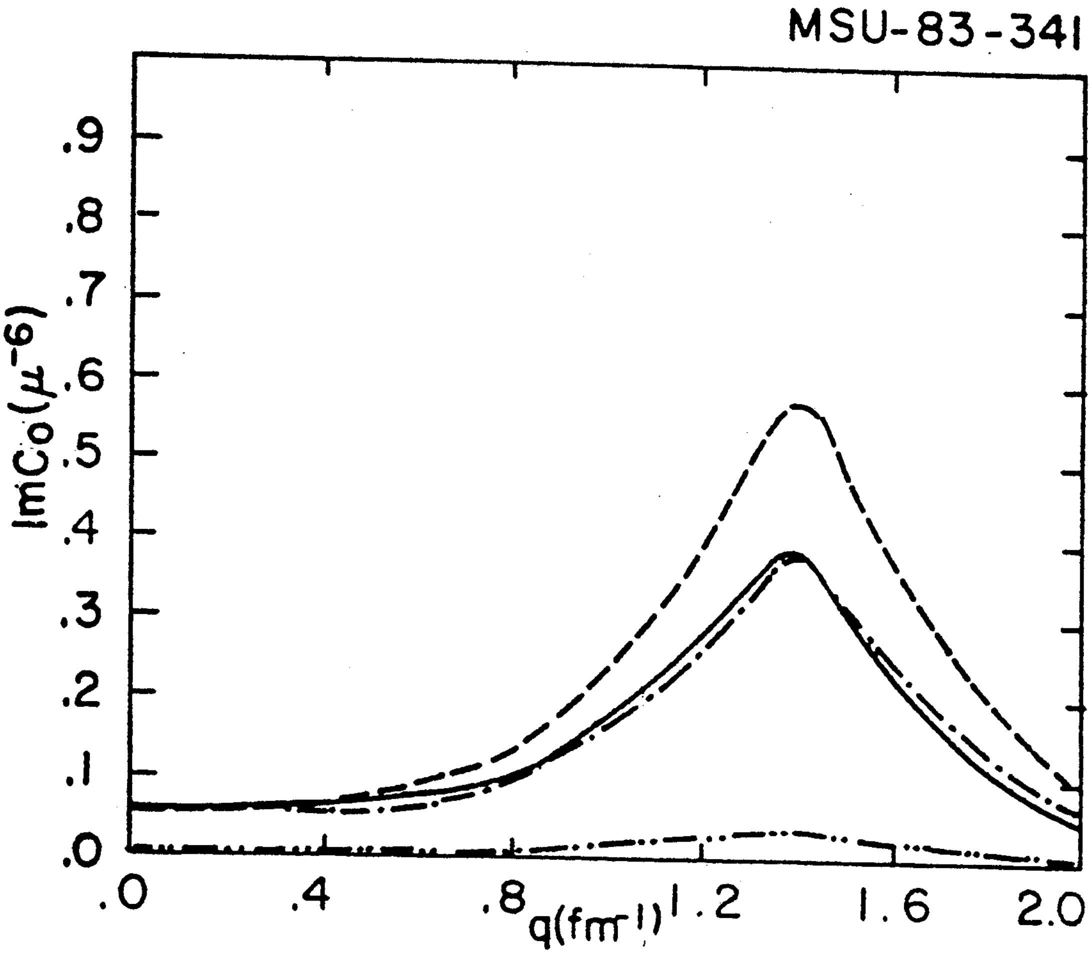


Figure V.4

The imaginary component of the two-body P wave pion absorption parameter of nuclear matter vs. pion momentum.
Dashed line corresponds to one π exchange with Λ_π = 800 Mev.
Solid line corresponds to $\pi + \rho$ exchange with Λ_π = 1200 Mev and Λ_ρ = 2000 Mev (standard set).
Dashed-dot line corresponds to one π exchange with Λ_π = 700 Mev.
Dashed-dot-dot line corresponds to one π exchange with Λ_π = 400 Mev.

Figure V.5

The imaginary component of the P wave pion absorption parameter in nuclear matter vs. the cut off radius.
Dashed-dot-dot line corresponds to one π exchange with Λ_{π} = 1200 Mev .
Dashed line corresponds to one π exchange with Λ_{π} = 800 Mev .
Solid line corresponds to $\pi + \rho$ exchange with Λ_{π} = 1200 Mev and Λ_{ρ} = 2000 Mev (standard set).
Dashed-dot line corresponds to one π exchange with Λ_{π} = 700 Mev .

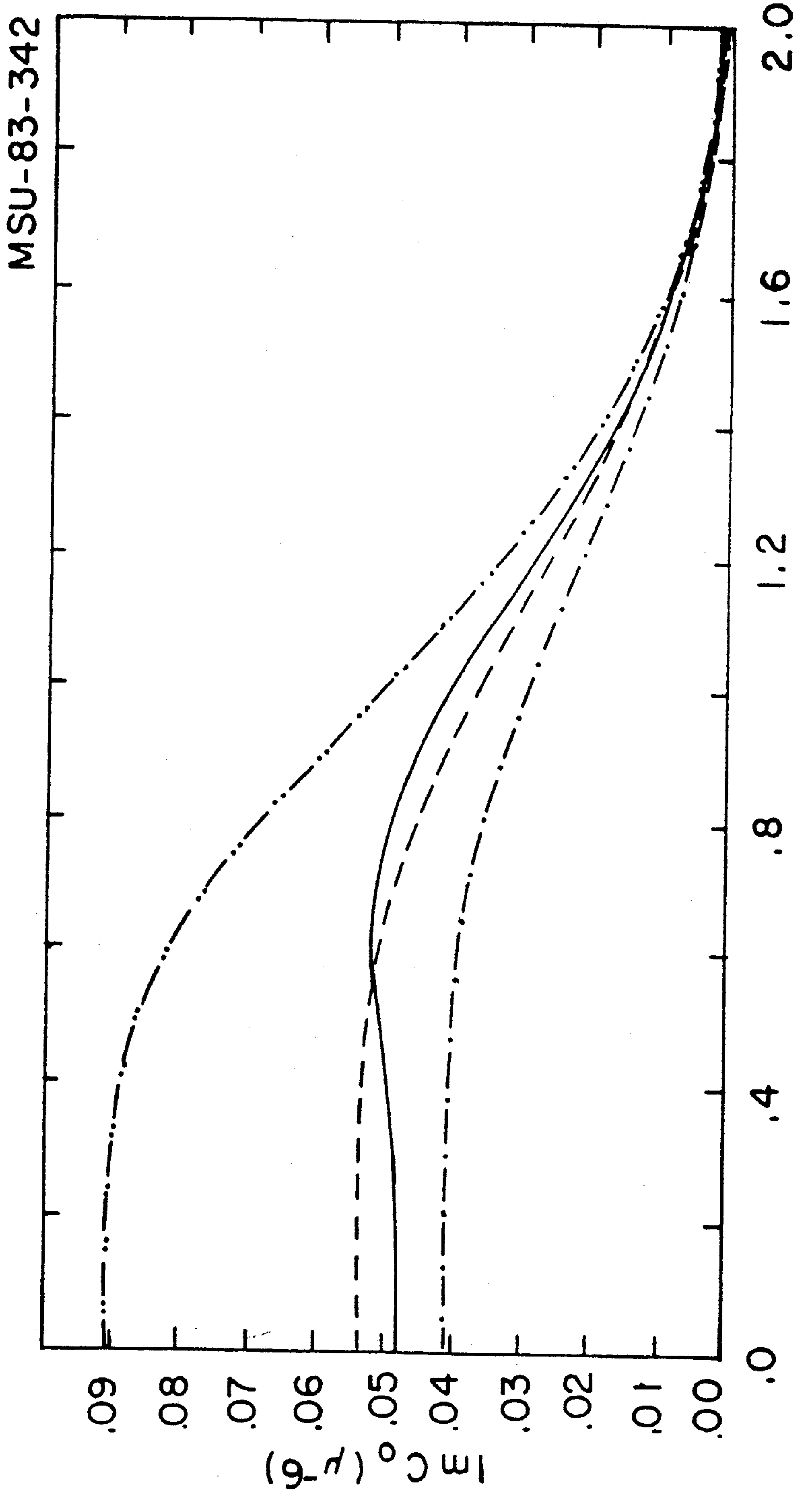


Figure V.5

concerned , the standard model is consistant with the little bag, the calculation with π- exchange only is consistant with the cloudy bag model, but elimination of the ρ may not be consistant with other nuclear physics phenomenology. In particular it is not consistant with S wave absorption, where Λ_π = 1200 Mev is required to get any appreciable absorption. If the M.I.T. bag size is correct, then the parameters of the model have no direct physical meaning, they simply are a parameterization of a non-local interaction in local form.

The question of bag dynamics has not been directly addressed by any one in this context. The calculations of Kisslinger and Miller [Ref. V.5] assume absorption of the pion by a nucleon to form a Δ , which then coaleses with a nucleon to form a six-quark bag. But as we have seen, formation of the Δ fixes the quantum numbers of the intermediate state, in this case the 6 quark bag, so that everything is undistinguishable from the standard model, the only difference is the calculation of the rate of the reaction $\Delta + N \rightarrow N + N$ via a 6 quark bag. This just changes the absolute scale and is parametrized anyway.

In conclusion calculations of pion absorption coefficients in nuclei, using the standard " $\pi + \rho$ " two-nucleon rescattering model with parameters fixed from πd absorption gives results in reasonable agreement with isotopic ratio and angular distribution of emitted nucleons. To get the magnitudes right, nuclear medium effects on the pion propagator must be taken into account. The theory is not complete as higher order processes have been shown to be important. From the fundamental piont of view, the model is consistant with the " little bag " model. It might be consistant with the cloudy bag model if another mechanism could be found for S wave absorption. If the M.I.T. bag model is correct, the model is purely parametric, containing no reference to fundamental processes.

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Appendix A (REPRINTS)

ISOSPIN DEPENDENCE OF PION ABSORPTION BY NUCLEON PAIRS IN THE He ISOTOPES

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We calculate the relative absorption ratio of a pion by $T = 0$ and $T = 1$ nucleon pairs in the He isotopes measured recently by Ashery et al. Standard theory based on Δ–isobar intermediate excitations agrees with the experimental observation that for energies around the resonance, pion absorption by a $T = 1$ nucleon pair is strongly suppressed.

Very recently Ashery et al. [1] performed the $(\pi^+, 2p)$ and (π^-, pn) measurements on ^{3}He and ^{4}He at pion kinetic energy of $T_\pi = 165$ MeV in order to obtain information on the cross sections for pion absorption by $T = 0$ and $T = 1$ nucleon pairs. They found surprisingly large ratios for $d\sigma(\pi^+, pp)/d\sigma(\pi^-, pn)$, on the order of ~100. By counting the numbers of initial nucleon pairs with the isospin $T = 0$ and $T = 1$ in those nuclei, these ratios were expressed in terms of the isospin of initial nucleon pairs. It was then found that $R \equiv d\sigma(T = 0)/d\sigma(T = 1) \approx 50$ [1]. If one estimates this ratio in terms of the isospin geometry for the formation of Δ isobars, one finds $R = 2$ [1,2].

In this short note, we would like to demonstrate that our current understanding of the pion nucleon interaction and the Δ isobar formation provides a natural explanation for the observed large value of this ratio.

In recent years, there have been a number of theoretical derivations of the pion–nucleus optical potential parameters from the pion–nuclear many body theory [3–11]. In particular, the imaginary part of the two body optical potential has a direct relationship to pion absorption. The S-wave and P-wave pion absorption mechanisms seem to provide a microscopic understanding of the optical parameters derived from pionic atoms and pion–nucleus elastic scattering cross sections [12]. The S-wave pion absorption mechanism accounts for the experimental fall off of the $\pi d \to pp$ cross section with increasing pion energy and becomes negligibly small above $T_\pi \approx 50$ MeV. On the other hand, the P-wave pion absorption mechanism, in particular through the Δ isobar intermediate excitation, becomes dominant above $T_\pi \approx 100$ MeV.

Since our interest is in pion absorption in the resonance energy region, we concentrate on the P-wave absorption process through the Δ isobar intermediate excitation alone, as depicted in fig. 1. The wavy lines denote the spin–isospin dependent interaction which is usually described in terms of the pion and rho meson exchanges [13]. The T-matrix for pion absorption by the pion exchange mechanism is obtained by the Feynman graph technique in momentum space;

$$\begin{aligned} T(\boldsymbol{k}_1, \boldsymbol{k}_2; \boldsymbol{k}) = {} & [f(k_1)/\mu](\boldsymbol{\sigma}_1 \cdot \boldsymbol{k}_1)\boldsymbol{\tau}_1 \cdot D_\pi(\boldsymbol{k}_1) \\ & \times [f^*(k_1)/\mu](\boldsymbol{S}_2^+ \cdot \boldsymbol{k}_1) \cdot \boldsymbol{T}_2^+ G_D(P_\Delta)[f(k)/\mu]\, \boldsymbol{S}_2 \cdot \boldsymbol{k} T_{2\lambda} \\ & + [f(k_1)/\mu](\boldsymbol{\sigma}_1 \cdot \boldsymbol{k}_1)\boldsymbol{\tau}_1 \cdot D_\pi(\boldsymbol{k}_1) \\ & \times [f^*(k_1)/\mu](\boldsymbol{S}_2 \cdot \boldsymbol{k}_1) \cdot \boldsymbol{T}_2 G_C(P_\Delta) \\ & \times [f(k)/\mu]\, \boldsymbol{S}_2^+ \cdot \boldsymbol{k} T_{2\lambda} + (1 \rightleftharpoons 2). \end{aligned} \quad (1)$$

Here $\boldsymbol{k}$ is the incoming pion momentum in the laboratory frame, which after absorption gets distributed as $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ among the two nucleons 1 and 2 involved in the process. $\boldsymbol{\sigma}, \boldsymbol{\tau}$ are the spin, isospin Pauli matrices and $\boldsymbol{S}, \boldsymbol{T}$ the corresponding transition operators between a nucleon and a Δ isobar. The $\pi N \Delta$ coupling

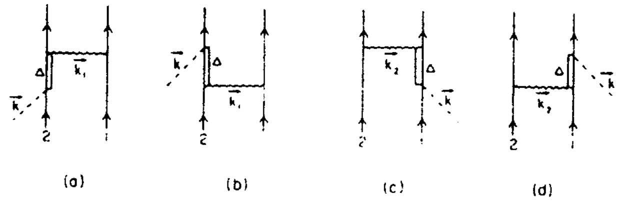

Fig. 1. P-wave pion absorption mechanism with Δ isobar intermediate excitation.

constant f^* is related to the πNN coupling constant f through $f^* = 2f$ (Chew–Low relation) and has the monopole form factor $f^*(k) = [(\Lambda^2 - \mu^2)/(\Lambda^2 - k^2)]\, f^*$, where Λ is the cutoff mass. D_π is the pion propagator and μ the pion mass. The Δ isobar propagators for the direct (figs. 1a and 1c) and the crossed (figs. 1b and 1d) graphs are

$$G_D(P_\Delta)^{-1} = m_\Delta - m + P_\Delta^2/2m_\Delta - \omega - \tfrac{1}{2}\mathrm{i}\Gamma,$$

$$G_C(P_\Delta)^{-1} = m_\Delta - m + P_\Delta^2/2m_\Delta + \omega, \tag{2}$$

where the Δ isobar mass and nucleon mass are denoted by m_Δ and m, and the pion energy by ω. We take the static nucleon approximation and therefore $|P_\Delta|^2 = |k|^2$ for G_Δ and $|P_\Delta|^2 = m\omega$ for G_C. For the width of the Δ isobar, we take the empirical relation [14].

$$\Gamma = \{0.47/[1 + 0.6(q/\mu)^2]\}\, q^3/\mu^2. \tag{3}$$

The pion–nucleon center-of-mass momentum q is related to the incoming pion momentum through $q = (m/\sqrt{s})\, k$, where s is the s-channel invariant mass.

The T-matrix [eq. (1)] together with a similar expression for the rho meson exchange process can be further worked out using the technique developed in ref. [6]. A lengthy expression is then obtained for pion absorption in terms of two body transition operators in the nucleon space, the details of which will be published elsewhere [15].

In principle, many pion partial waves contribute to the absorption cross section; l_π(pion partial wave) $\lesssim k \cdot R$ (nuclear radius). However, for the sake of discussion let us take the dominant partial wave $l_\pi = 1$, absorption of which by a 1S-orbit nucleon leaves the intermediate Δ isobar in the 1S-orbit [1]. In this case, the ratio R does not depend on any details of the model parameters and is simply given by the ratio of the squares of the Δ–isobar propagators G_D and G_C of eq. (2);

$$R = \mathrm{d}\sigma(T = 0)/\mathrm{d}\sigma(T = 1) = |G_D/G_C|^2. \tag{4}$$

The calculated ratio using eq. (4) is shown by the dashed line in fig. 2. Because of the resonant behavior in the direct channel, the ratio has a peak at $k \approx 2\mu$ ($T_\pi \approx 180$ MeV). At the experimental energy, $T_\pi = 165$ MeV [1], this ratio is ~200. This behavior is understood as follows. While $l_\pi = 1$ pion absorption by a $T = 0$ nucleon pair is allowed by the direct absorption mechanism (figs. 1a and 1c), this is forbidden for a $T = 1$ nucleon pair. For the latter case, absorption of a $l_\pi = 1$ pion by a $T = 1$ pair ($S = 0$, $L = 0$) leads to $T = 1$, $J^\pi = 1^+$ intermediate states. Those quantum numbers are not allowed by the Pauli principle for the final nucleon pair. Instead, pion absorption by a $T = 1$ pair proceeds through the crossed absorption mechanism (figs. 1b and 1d) with exactly the same matrix elements as the direct $T = 0$ absorption process except the Δ isobar propagators.

A question to examine now is if other partial wave contributions in particular for $T = 1$ pion absorption are small enough to keep the ratio sufficiently large. One thing to note in this respect is that absorption of $l_\pi \neq 1$ forces the intermediate Δ isobar to move with a finite angular momentum ($l_\Delta \neq 0$). This fact, together with the large momentum transfer ($\Delta q \sim \sqrt{m\omega}$) between the two nucleons required for pion absorption [16] (short range process) tends to cut down to a great extent the spatial transition matrix between the initial and final states.

In the pion absorption operator, there is spin–spin interaction as well as the tensor interaction [6]. The large momentum transfer requirement forces two nucleons to be in the short distance ($r \lesssim 0.5$ fm), whereas the strong short range correlations prevent them from coming together. This interplay makes the contribution of the spin–spin interaction negligibly small [16]. Therefore, we shall take only the tensor interaction terms in our calculation. Furthermore, the ratio R weakly depends on the angle between the outgoing nucleons and we choose the angle such that the relative momentum of the nucleon pair is perpendicular to the incoming pion momentum. The results with pion exchange alone with a cutoff mass of $\Lambda_\pi = 0.8$ GeV are shown in table 1. When this value of Λ_π is used in the π exchange model, it gives results similar to those of the standard ($\pi + \rho$) exchange model, where a larger value of Λ_π is used [16]. The results on the ratio are

Table 1
Pion absorption cross sections in several channels as a function of pion momentum k are compared, where l_π is the angular momentum of the incoming pion. These numbers are normalized to the $(T, S, L) = (0, 1, 0)$ to $(T', S', L') = (1, 0, 2)$ transition at $k = 0.5\ \mu$ as indicated by *.

Initial	l_π	T'	S'	L'	σ				
					$k[\mu] = 0.5$	1.0	1.5	2.0	2.5
$T=0, S=1, L=0$	0	1	1	1	0.027	0.40	2.9	9.7	7.3
	1	1	0	2	1*	4.9	21	52	30
	2	1	1	3	0.006	0.11	1.0	4.2	3.6
$T=1, S=0, L=0$	0	1	1	1	0.008	0.086	0.44	1.1	0.63
	1	0	1	2	0.076	0.17	0.24	0.26	0.26
	2	1	1	3	0.002	0.023	0.14	0.40	0.26

not very sensitive to the choice of Λ_π within the reasonable range.

We show in table 1 the pion absorption cross sections in several channels for $l_\pi \leqslant 2$ as a function of pion momentum k. The $l_\pi = 0$ and 2 contributions are indeed very small as compared to the dominant one ($l_\pi = 1, T = 0 \to T' = 1$) even up to large momenta considered here particularly for the initial isospin $T = 1$ case. This is due to the small spatial matrix elements for $l_\pi = 0$ and 2 as compared to those for $l_\pi = 1$, typically about 1/5 in the resonance region, and strong cancellation between the two competing terms for $T = 1$. Note that the explicit distinction between the direct and crossed Δ propagators G_D and G_C, which was neglected by Ko and Riska [6], is important for this cancellation. The higher partial wave contributions ($l_\pi \geqslant 3$) are smaller by an order of magnitude. The summed values for the $T = 0$ and $T = 1$ initial paris are then used to derive the ratio R as a function of the pion momentum, which is depicted by the solid line in fig. 2. The ratio is now down to ≈ 40 at $T_\pi = 165$ MeV. The experimental results taken at different angles are also shown with error bars [1].

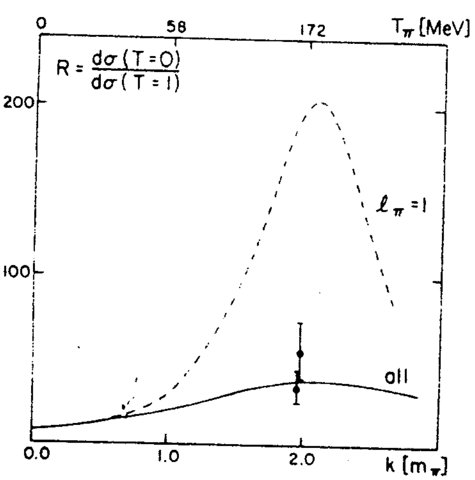

Fig. 2. The pion absorption ratio $R = d\sigma(T = 0)/d\sigma(T = 1)$ as a function of the pion momentum k (T_π = kinetic energy). The dashed line denotes the $l_\pi = 1$ contribution only [eq. (4)], whereas the solid line includes all. The experimental data taken at different angles at $T_\pi = 165$ MeV are depicted by the points with error bars.

Finally as stated at the beginning, the S-wave pion absorption process ($l_\pi = 0$) is important at low pion energies. Therefore, the calculated results have to be used with caution below $T_\pi \approx 100$ MeV. The capture of pions from the 2p level in pionic atoms [17] might, however, be compared with the present result at $T_\pi = 0$ MeV [6,10]. Furthermore, we have been neglecting the P-wave absorption process through nucleon intermediate states. This is certainly small for $T = 0$ pion absorption [16], but might give an additional contribution for absorption by a $T = 1$ pair.

In conclusion, we have demonstrated that our knowledge of the pion–nucleon interaction and the Δ isobar formation gives a natural explanation for the large ratio of the pion absorption cross sections by $T = 0$ and $T = 1$ nucleon pairs in the resonance region. The resulting ratio $R = d\sigma(T = 0)/d\sigma(T = 1)$ shows only a slight resonance behavior. It would be interesting to measure this energy dependence and also the angular distributions for a $T = 1$ pair, which should show an interesting variation with energy due to the interplay between the three ($l_\pi = 0 \div 2$) partial waves.

We are extremely grateful to G.F. Bertsch for introducing us to these interesting experimental data and for enlightening discussions in the course of the work. Thanks are also due to G.E. Brown and W. Weise for stimulating discussions and to D. Ashery, E. Piasetzhy and J.P. Schiffer for experimental information. This work was supported by the National Science Foundation under grant no. PHY-80-17605.

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THE EFFECT OF THE NUCLEAR MEDIUM ON S-WAVE PION ABSORPTION ☆

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It is shown that the effects of the nuclear medium on the pion propagator and the renormalization of the πNN interaction, described in terms of excitation of virtual isobar–hole pairs can enhance the predicted absorption rates for pions at threshold to values close to the empirical ones.

Recent attempts at explaining the empirical rates for nuclear pion absorption at threshold in terms of simple two-body rescattering mechanisms have led to underpredictions of the order 20–30% [1]. Similarly the predicted values for the main absorptive two-nucleon component of the S-wave pion-nucleus optical potential have also been considerably smaller than those suggested by optical model studies of pion atom data [1–4]. We shall here show that the effects of successive excitation of virtual isobar–hole pairs in the nuclear medium on the propagation of the rescattered meson, and on the P-wave πNN interaction, can enhance the predicted absorption rates by the amounts needed.

The absorption of S-wave pions at threshold is usually described in terms of the two-body rescattering mechanism illustrated in fig. 1a. Here the pion undergoes an initial S-wave scattering off one nucleon, and then propagates through the nucleus until absorbed on a second nucleon by the usual P-wave πNN interaction. The presence of the nuclear medium adds a self energy Π to the pion propagator and renormalizes the final P-wave interaction. These medium corrections are schematically indicated in fig. 1b.

For the pion self energy we employ the picture of Barshay, Brown and Rho [5] and Baym and Brown [6] which involves excitation of isobar–hole pairs. We shall use a similar description to obtain the πNN vertex renormalization, following the suggestion of Rho [7]. In this picture the pion self energy is made up of the single isobar hole pairs connected by the reduced isobar–hole interaction (R), with the single pion exchange contribution subtracted out, as illustrated in fig. 2.

The self energy contribution from the single isobar–hole pair is, with account of both time orderings of the Δ_{33} resonance,

$$\Pi_0 = -\frac{4}{9}\left(\frac{f_\Delta}{\mu}\right)^2 k^2 f_\pi^2(k)\rho\left(\frac{1}{D_1}+\frac{1}{D_2}\right) . \qquad (1)$$

Here k is the pion momentum, μ the pion mass and ρ

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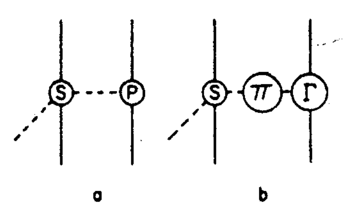

Fig. 1. S-wave pion absorption mechanism without (a) and with (b) medium effects.

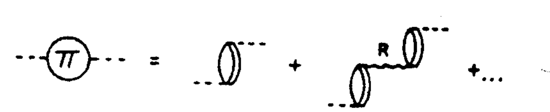

Fig. 2. Model for the pion self energy Π involving the reduced isobar–hole interaction R.

the nuclear density. In (1) f_Δ is the usual $\pi N\Delta$ coupling constant ($f_\Delta^2/4\pi \sim 0.32$–$0.35$) [8] and $f_\pi(k)$ the form factor associated with the $\pi N\Delta$ vertex. Finally the isobar energy denominators D are, with the pion energy being $\mu/2$ for threshold absorption,

$$D_1 = m_\Delta - m - \mu/2\,, \quad D_2 = m_\Delta - m + \mu/2\,, \tag{2}$$

and with m_Δ and m being the nucleon and Δ_{33}-resonance masses respectively. Because of the low energy of the virtual pion we have not included self energy corrections to the isobar propagators in eq. (2). Although the free width of the isobar vanishes below threshold an additional width in the nuclear medium can arise from coupling to P-wave absorptive channels below threshold.

The complete self energy is obtained by summing the the chain of isobar–hole diagrams connected by the reduced isobar–hole interaction (R) indicated in fig. 2. The result may be expressed in the general form

$$\Pi(k) = \Pi_0(k)\,\mathrm{Tr}\left\{\hat{k}^{\mathrm{T}}\,\frac{1}{1-[\Pi_0(k)/k^2 f_\pi^2(k)]R(k)}\,\hat{k}\right\}, \tag{3}$$

where $R_{mn}(k)$ is the isobar–hole interaction in the isospin 1 channel with the single pion exchange contribution removed, and multiplied by $(f_\Delta/\mu)^{-2}$ (following the notation of Weise [9]).

The model we consider for the πNN vertex in the nuclear medium is based on the isobar–hole chain diagrams illustrated in fig. 3. The final isobar–hole pair is connected with the active nucleon by the reduced isobar–hole – particle–hole (Δh–ph) interaction in the isospin 1 channel R', from which the single pion exchange contribution has been subtracted. The other isobar–hole pairs are connected by the reduced isobar–hole interaction R defined above. We define R' so that the explicit pion-exchange strength factor ff_Δ/μ^2 has been divided out in analogy with Weise's definition of R [9] (f is the pseudovector πN coupling constant).

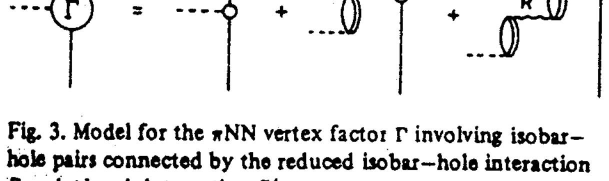

Fig. 3. Model for the πNN vertex factor Γ involving isobar–hole pairs connected by the reduced isobar–hole interaction R and Δh–ph interaction R'.

With that definition, the medium renormalized πNN vertex factor may be written as

$$\Gamma(k) = \mathrm{Tr}\left\{\hat{k}^{\mathrm{T}}\,\frac{1+[\Pi_0(k)/k^2 f_\pi^2(k)][R'(k)-R(k)]}{1-[\Pi_0(k)/k^2 f_\pi^2(k)]R(k)}\,\hat{k}\right\}. \tag{4}$$

We have not included here the exchange diagrams for the vertex normalization effect. These should be expected to be small according to the discussion in ref. [7].

With the results (3) and (4) for the self energy and the renormalized vertex factor $\Gamma(k)$ for the πNN interaction we can now consider the effects of the medium on the S-wave absorption process illustrated in fig. 1. Assuming that the initial S-wave rescattering interaction can be described by a phenomenological zero-range hamiltonian, the two-body rescattering amplitude becomes

$$T = -\mathrm{i}\,\frac{8\pi}{\mu^{5/2}}\,f\,\frac{\Gamma(k) f_\pi^2(k)}{\mu^2+k^2-\omega^2+\Pi(k)}\,\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{k}$$
$$\times\left[\lambda_1\tau_a^2 - \mathrm{i}\,\frac{\lambda_2}{2}\left(1+\frac{\omega}{\mu}\right)(\tau^1\times\tau^2)_a\right]. \tag{5}$$

In this expression λ_1 and λ_2 are the two coupling constants for the S-wave hamiltonian [10] which have the values $\lambda_1 = 0.003$ and $\lambda_2 = 0.050$ as determined from the S-wave πN phase shifts.

The simplicity of the expression (5) allows one to calculate the absorption rates for pions in nuclei at threshold using the expressions given in ref. [1] for the case $\Pi = 0$, $\Pi = 1$, with the simple replacement of the Fourier transform of the pion propagator (Y_1) in the radial matrix element expressions with the more general expression:

$$\mu^{*2} Y_1(\mu^* r) \to \frac{2}{\pi}\int_0^\infty \mathrm{d}k\,\frac{k^3 j_1(kr)\,\Gamma(k) f_\pi^2(k)}{k^2+\frac{3}{4}\mu^2+\Pi(k)}\,. \tag{6}$$

To proceed we need explicit models for the isobar–hole interactions $R(k)$ and $R'(k)$ in eqs. (3) and (4). The simplest model is obtained by considering the interactions made up solely by pion and ρ-meson exchange. If the short range correlations between the successive isobar–hole lines are taken into account to the minimal extent that they eliminate the δ-function components in the meson exchange interactions, one obtains for R the result [9]

$$R_{mn}(k) = \tfrac{1}{3}\delta_{mn}\,[f_\pi^2(k) - 2C_\rho f_\rho^2(k)]\,, \tag{7}$$

where $f_\rho(k)$ is the form factor associated with the $\rho N\Delta$ vertex and C_ρ the strength factor for the ρ-meson exchange interaction:

$$C_\rho = (g_{\rho\Delta}/2m)^2(\mu/f_\Delta)^2 . \quad (8)$$

Here $g_{\rho\Delta}$ is the $\rho N\Delta$ coupling constant [8]. With the same "minimal" account of the short range correlations the reduced Δh–ph interaction R' can be cast into the form

$$R'_{mn}(k) = \tfrac{1}{3}\delta_{mn}\,[f_\pi^2(k) + 2C'_\rho f_\rho^2(k)] , \quad (9)$$

with the ρ-meson exchange coefficient

$$C'_\rho = \left(\frac{g_{\rho\Delta}g_{\rho N}(1+\kappa)}{4m^2}\right)\left(\frac{\mu^2}{ff_\Delta}\right) . \quad (10)$$

Here $g_{\rho N}$ is the vector ρ-nucleon coupling constant and κ the tensor coupling strength.

The first (pion) terms in these expressions for R and R' represent the classical Lorentz–Lorenz–Ericson–Ericson effect [11] as derived by Barshay, Brown and Rho [5] for the self energy and by Rho [7] for the vertex renormalization. It was shown in ref. [9] that a more accurate treatment of the nuclear pair correlation function reduces these pion terms to a fraction of the value in eqs. (7) and (8), whereas the ρ-meson terms are only reduced by a moderate amount. Accordingly we shall neglect the pion terms in R and R' and incorporate the finite range corrections approximately into $f_\rho(k)$.

Note now that

$$\frac{C'_\rho}{C_\rho} = \frac{g_{\rho N}(1+\kappa)/g_{\rho\Delta}}{f/f_\Delta} . \quad (11)$$

This ratio is unity if the ratio of the $\pi N\Delta$ to πNN coupling constants is the same as the ratio for the $\rho N\Delta$ to ρNN coupling constants as predicted in the static quark model. In that case $R' = R$ and thus the numerator correction factor in eq. (4) for R vanishes, and the simple vertex renormalization model of Rho [7] results. We shall use that simpler model here with the justification that recent studies of the resonant rescattering contributions in the reaction $\pi^+ d \leftrightarrow pp$ indicate that the π and ρ coupling constant ratios should be close [12,13]. In the quark model [8,9]

$$C_\rho = C'_\rho = (g_{\rho N}/2m)^2(1+\kappa)^2(\mu/f)^2 \approx 2 , \quad (12)$$

with the values given in ref. [14] for the ρ-nucleon coupling constants.

Using these results and the substitution (6) we have used the formula given in ref. [1] to calculate the parameter $\mathrm{Im}\,B_0$ for the two-body absorptive part of the S-wave pion-nucleus optical potential. Taking the form factors $f_\pi(k)$ and $f_\rho(k)$ to have the usual monopole form

$$f(k) = \frac{\Lambda^2 - \bar{m}^2}{\Lambda^2 + k^2 - \omega^2} , \quad (13)$$

with $\bar{m}$ being the mass of the exchanged meson and $\Lambda = 1.2$ GeV/c^2 we obtain $\mathrm{Im}\,B_0 \simeq 0.048\,\mu^{-4}$ in reasonable agreement with the empirical value $0.042\,\mu^{-4}$. Note that with no medium corrections the corresponding result would be $0.020\,\mu^{-4}$ [2], which is too small by a factor of 2. The value $\Lambda = 1.2$ GeV/c^2 for the pion form factor is the same that has been used in studies of $\pi^+ d \to pp$ [12]. On the other hand, in studies of $\pi^+ d \to pp$, Λ is usually taken to be much larger than 1.2 GeV/c^2 for the ρ-meson vertices [12,13]. The use of the smaller value here simply serves to mock up neglected finite range corrections to the nuclear pair correlation function as mentioned above. If Λ were taken to be 2.0 GeV/c^2 for the ρ-meson form factor we would obtain the far too small value $0.016\,\mu^{-4}$ for $\mathrm{Im}\,B_0$!

The values for the residual isobar–hole interaction R given in eq. (7) were obtained with the schematic model of a step function for the nuclear pair correlation function. Using a more realistic model for the pair correlation function, Weise [9] found that the whole denominator factor in eqs. (3) and (4) could be well approximated by an almost momentum-independent expression of the form

$$1 - \frac{\lambda}{3}\frac{\Pi_0(k)}{k^2}\frac{1}{f_\pi^2} , \quad (14)$$

with $1 < \lambda < 2$, depending on the specific details of the correlation function. In fig. 4 we show $\mathrm{Im}\,B_0$ as calculated with this simpler model as a function of the parameter λ (neglecting the pion form factor). Clearly $\mathrm{Im}\,B_0$ is a sensitive function of λ. The desired value $0.042\mu^{-4}$ for $\mathrm{Im}\,B_0$ is obtained with $\lambda = 1.7$, which agrees well with the "optimal" value $\lambda = 1.6$ found in recent comprehensive optical model studies of pion-nucleus scattering data [15]. (If the pion form factor (13) were included in the self energy the desired value for $\mathrm{Im}\,B_0$ is obtained with $\lambda = 1.5$.) The fact that the absorptive optical potential may be explained with a similar value for the Lorenz–Lorentz factor λ as used in studies of

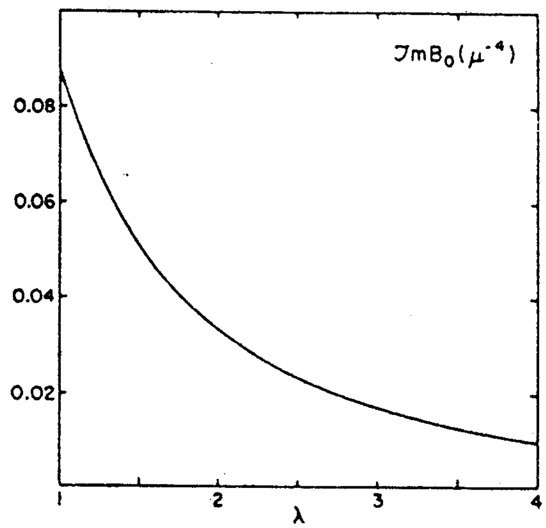

Fig. 4. The parameter $\mathrm{Im}\,B_0$ describing the strength of the ρ^2 term in the pion-nucleus S-wave optical potential.

other aspects of the pion-nucleus interaction should, in our opinion, be viewed as strong support of the validity of the present picture of the medium effects.

The combined medium effects on the pion propagator and πNN vertex enhance the predicted value for $\mathrm{Im}\,B_0$ by roughly a factor 2. As this parameter represents the absorption rate in nuclear matter, the medium corrections will be much smaller in light nuclei, which have large densities only in the central region. To illustrate this we consider the absorption rate in ^{4}He at threshold. We calculate this as an average over the nucleon density $\rho(r)$ for the ^{4}He nucleus

$$\Gamma = \int d^3r\, \rho(r)\, \Gamma(\rho)\,, \qquad (15)$$

with $\Gamma(\rho)$ being the result obtained by replacing the pion exchange Yukawa function in the explicit expression for Γ given in ref. [1] by the medium corrected form (6). Using the harmonic oscillator model in (15) we find that the medium corrections enhance the calculated absorption rate by $\sim$40% for the α-particle if λ in eq. (14) is taken to be 1.7. Depending on the value of Λ in the pion form factor (13) the imaginary part of the scattering length for ^{4}He falls between $0.017\,\mu^{-1}$ ($\Lambda_\pi = 1.2$ GeV) and $0.034\,\mu^{-1}$ ($\Lambda_\pi = \infty$) which should be compared to $\mathrm{Im}\,a(\exp) \simeq 0.03\,\mu^{-1}$. Without the medium effects the corresponding range of the calculated scattering length would be $0.012\,\mu^{-4} < \mathrm{Im}\,a < 0.021\,\mu^{-4}$, which demonstrates the need for the medium effect.

In conclusion we have demonstrated that the medium corrections to the pion propagator and πNN vertex in nuclei enhance the predicted absorption rates by roughly the amount needed to explain the values extracted from pionic atom data. The pion self energy by itself induces a far too large enhancement which is moderated by the vertex renormalization. Both effects must therefore be considered together.

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Effective S-wave πNN interaction in nuclei

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We show that virtual pion rescattering in a nuclear medium gives rise to an effective S-wave πNN interaction Hamiltonian. The first few terms in the density expansion of the strength parameter are estimated and shown to be appreciable in nuclear matter.

[NUCLEAR REACTIONS Pion-nucleus interactions: S-wave interaction.]

Considerable, if inconclusive, attention has been given in recent literature to the "Barnhill" ambiguity in the S-wave πNN interaction in nuclei.[1-5] The ambiguity concerns the strength parameter λ in the Hamiltonian for this interaction:

$$H=\lambda \frac{f}{\mu}\psi^\dagger\vec{\sigma}\cdot\nabla_N\vec{\tau}\cdot\vec{\phi}\psi . \quad (1)$$

Here ψ and $\vec{\phi}$ are the nucleon and isovector pion field operators and ∇_N a symmetrized gradient operator acting on the nucleon fields.[1] In (1) f is the pseudovector pion-nucleon coupling constant ($f^2/4\pi = 0.08$) and μ the pion mass.

Straightforward nonrelativistic reduction of the Lorentz invariant pseudovector πNN interaction Hamiltonian leads to $\lambda=\mu/2m$ (m = nucleon mass). The unitary freedom inherent in this reduction leads to the ambiguity in λ.[1] The physical reason for this ambiguity is the lack of a definite treatment of the binding effects in a nuclear medium that force the nucleon under consideration off shell.[3,4] Only in simplified boson exchange models is it possible to make definite predictions for λ.[6] Therefore it has been suggested that the parameter be determined empirically by means of (p,π^+) and (π^+,p) reactions interpreted with a single nucleon stripping model.[3,7] Apart from the obvious criticism that the simple stripping model may not be adequate, we shall in this work show that sizeable density dependent medium corrections to the effective one-body operator (1) are caused by virtual pion rescattering. Thus λ will not have any universal value valid for a range of nuclei, and attempts at empirical determination of λ will be futile.

The main rescattering process that contributes to λ is that involving two nucleons: an incident S-wave pion rescatters off a nucleon (which is ejected) and is absorbed by a particle-hole pair, or it rescatters off a particle-hole pair and is absorbed by a final nucleon (which is ejected). This is illustrated in the diagrams in Fig. 1, which include direct and exchange terms. These processes take place in addition to "true" two-nucleon absorption processes in which two nucleons are ejected from the nucleus.

The diagram in Fig. 1(c) represents distortion of the incident pion wave function and should not be included in the basic πNN interaction, as the distortion can be treated with an optical potential. The corresponding exchange term in Fig. 1(d) should be excluded for the same reason. The amplitude for the diagram in Fig. 2(a) vanishes in spin or isospin 0 nuclei because of the spin-vector isovector nature of the πNN absorption operator. Therefore only the remaining exchange term diagram [Fig. 1(b)] contributes to the S-wave absorption interaction.

In order to construct the amplitude corresponding to Fig. 1(b) we employ the phenomenological zero-range Hamiltonian

$$H=4\pi\frac{\lambda_1}{\mu}\bar{\psi}\vec{\phi}\cdot\vec{\phi}\psi+4\pi\frac{\lambda_2}{\mu^2}\bar{\psi}\vec{\tau}\cdot\vec{\phi}\times\vec{\pi}\psi \quad (2)$$

to describe the S-wave rescattering vertex ($\vec{\pi}=\partial_0\vec{\phi}$). In (2) the coupling constants λ are de-

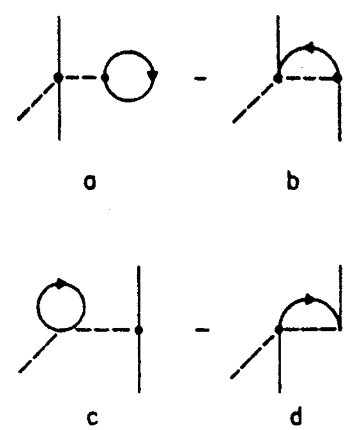

FIG. 1. Pion rescattering contributions to the effective S-wave πNN interaction.

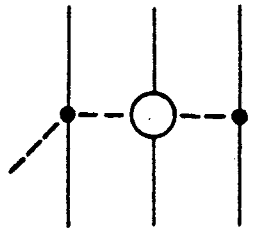

FIG. 2. Three-body contribution to pion absorption with double pion rescattering.

termined from the S-wave pion-nucleon phase shifts to be $\lambda_1 = 0.003$ and $\lambda_2 = 0.05$.[8] This Hamiltonian leads to a satisfactory prediction of the S-wave pion-nucleon phase shifts at low energies and is used in the standard derivations of the first and second order S-wave pion-nucleus optical potential. It has also been used successfully in the two-nucleon model for nuclear pion absorption,[9,10] although it implies a very simple direct off shell extrapolation of the pion-nucleon interaction. For the final absorption vertex we use the standard P-wave πNN interaction. The two-body rescattering amplitude is then

$$T=-4\pi i\left(\frac{f}{\mu^2}\right)\frac{\vec{\sigma}^2\cdot\vec{k}}{\mu^2+k^2}\left[2\tau_i^2\lambda_1-i\lambda_2(\vec{\tau}^1\times\vec{\tau}^2)_i\right], \quad (3)$$

with $\vec{k}$ being the momentum of the exchanged pion. The isospin index of the initial pion is i.

Taking the matrix element of T over the closed particle-hole line in Fig. 1(b) yields

$$T=i\frac{4}{\pi}\frac{f}{\mu^2}(\lambda_1+\lambda_2)k_F^3\int d^3r_2\frac{j_1(k_F r)}{k_F r}\times\int\frac{d^3k}{(2\pi)^3}\frac{\vec{\sigma}\cdot\vec{k}}{k^2+\mu^2}e^{-i\vec{r}\cdot\vec{k}}. \quad (4)$$

Here k_F is the Fermi momentum and $\vec{r}=\vec{r}_1-\vec{r}_2$. To reduce (4) to an effective one-body operator we take the 0-range limit $\mu\to\infty$, which leads to

$$T=-i\frac{2}{3\pi}\left(\frac{k_F}{\mu}\right)^3\left(\frac{f}{\mu}\right)(\lambda_1+\lambda_2)\vec{\sigma}\cdot(\vec{p}_f+\vec{p}_i)\tau_i. \quad (5)$$

Here p_i and p_f are the momenta of the initial and final nucleons, which appear after a partial integration. The employment of the zero-range limit can be expected to overestimate the two-body contribution in Eq. (4). It is, however, a better approximation than that which the static pion propagator in Eq. (4) would imply, because of the self-energy correction to the propagator which reduces the kinetic energy term. This point will be discussed in detail in conjunction with the higher order corrections below.

The amplitude (5) can be generated from the interaction Hamiltonian (1) with the density dependent parameter λ_{II}:

$$\lambda_{II}=\frac{\pi}{\mu^3}(\lambda_1+\lambda_2)\rho=\frac{2}{3\pi}\left(\frac{k_F}{\mu}\right)^3(\lambda_1+\lambda_2). \quad (6)$$

In nuclear matter $k_F \simeq 1.35\ \text{fm}^{-1}$ and thus $\lambda_{II} \simeq 0.08$, which is similar in magnitude to that of the usual one-body Galiliean invariance counter term ($\lambda_I = \mu/2m \simeq 0.07$) obtained from the relativistic pseudovector Hamiltonian.

The magnitude for λ_{II} given in Eq. (6) is clearly an overestimate, as the effect of hadronic form factors and nuclear wave function correlations will tend to reduce it. Although the simple zero-range approximation used above will not permit a realistic estimate of these effects, a first estimate of the form factor reduction is possible. Assuming the pion-nucleon vertices to be described by a monopole type form factor $(\Lambda^2-\mu^2)/(\Lambda^2+k^2)$ the expression (1) ought to be multiplied by a factor $(1-\mu^2/\Lambda^2)$. With $\Lambda\sim 1\ \text{GeV}/c^2$, this would lead to a ~5% reduction of the previous estimate for λ_{II} remains the same as that of λ_I.

Equation (6) represents the second term in a density expansion of the parameter λ. The next term involves a secondary pion rescattering, as illustrated in Fig. 2. Here the dominant contribution will come from the P-wave component in the second rescattering amplitude. As the nucleon pole terms in this amplitude represent binding corrections to the two-body amplitude above, we only consider the intermediate states containing the Δ_{33} resonance treated in the sharp resonance

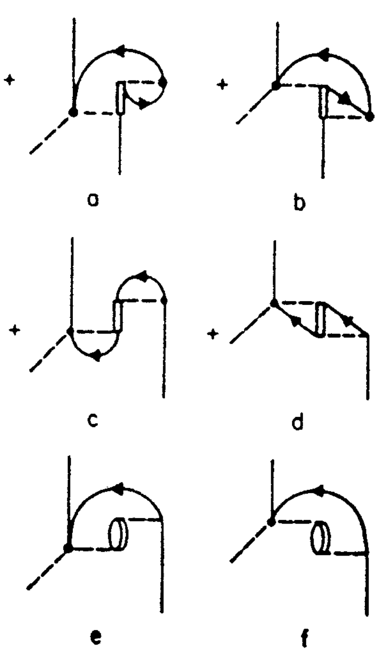

FIG. 3. Three-nucleon contributions with Δ_{33} intermediate states to the effective S-wave πNN interaction.

approximation.

Considering both time orderings for the Δ_{33} resonance intermediate state contributions there will then be a total of 36 three-body diagrams in which two of the final particles remain within the Fermi sea, compared to the four two-body diagrams in Fig. 1. After exclusion of all those diagrams that properly represent distortion of the incident pion wave function, and the corresponding exchange terms, and all those that give no contribution in isospin 0 or spin 0 nuclei, we only need to consider the six diagrams in Fig. 3.

To construct the scattering amplitudes for these diagrams we use the $\pi N\Delta$ coupling Lagrangian

$$\mathcal{L}_{\pi N\Delta}=\frac{f_\Delta}{\mu}\,\vec\chi^{\dagger}\vec\xi^{\dagger}\cdot\vec\nabla\vec\phi\,\xi\chi+\text{H.c.} \tag{7}$$

Here ξ and χ are the spin and isospin field operators for the nucleon and $\vec\xi$ and $\vec\chi$ the corresponding operators for the Δ_{33} resonance. From the decay width of the resonance one has $f_\Delta^2/4\pi\simeq0.35$. For the isobar contribution to the general three-nucleon two-pion exchange diagram in Fig. 2 one obtains

$$T=-4\pi i\,\frac{8}{9(m_\Delta-m)}\,\frac{ff_\Delta^{\,2}}{\mu^2}\,\frac{\vec\sigma^3\cdot\vec k_2}{(\vec k_1^{\,2}+\mu^2)(\vec k_2^{\,2}+\mu^2)}$$
$$\times\{\vec k_1\cdot\vec k_2[2\lambda_1\tau_i^{\,3}-i\lambda_2(\vec\tau^1\times\vec\tau^3)_i]-\tfrac14\vec\sigma^2\cdot(\vec k_2\times\vec k_1)$$
$$\times[2\lambda_1(\vec\tau^2\times\tau^3)_i-i\lambda_2(\vec\tau^1\times(\vec\tau^2\times\vec\tau^3))_i]\}, \tag{8}$$

with $\vec k_1$ and $\vec k_2$ being the momenta of the primary and secondary exchanged pions. The mass of the resonance is denoted by m_Δ.

Carrying out the spin and isospin sums over the closed particle-hole lines indicated in the diagrams in Figs. 3(a) and (b) and taking the Fourier transform of the amplitude (8) yields

$$T=-4\pi i\,\frac{8}{9(m_\Delta-m)}\,\frac{ff_\Delta^{\,2}}{\mu^2}\int\frac{d^3k_1}{(2\pi)^3}\,e^{i\vec k_1\cdot(\vec r_3-\vec r_2)}e^{i\vec k_2\cdot(\vec r_2-\vec r_1)}\,\frac{1}{k_1^{\,2}+\mu^2}\,\frac{1}{k_2^{\,2}+\mu^2}$$
$$\times(\vec\sigma^1\cdot\vec k_2\vec k_1\cdot\vec k_2-\tfrac13\vec k_2^{\,2}\vec\sigma^2\cdot\vec k_1+\tfrac43\vec k_2^{\,2}\vec\sigma^1\cdot\vec k_1). \tag{9}$$

The same sum for the diagrams in Figs. (3c) and (d) give rise to an identical result.

To complete the evaluation of the particle-hole line matrix elements for these diagrams [Figs. 3(a) and (b)] the expression (9) must be integrated over $\vec r_2$ and $\vec r_3$, folded with the relevant three-body density

$$\frac{k_F^{\,6}}{4\pi^4}\,\frac{j_1(k_Fr_{23})}{k_Fr_{23}}\,\frac{j_1(k_Fr_{13})}{k_Fr_{13}}\,. \tag{10}$$

The three-body density appropriate for the diagrams in Figs. 3(c) and (d) may be obtained from (10) by the replacement $\vec r_{13}\to\vec r_{12}$. Finally, because of nuclear correlations the δ function in the bracket in Eq. (9) should be eliminated by the replacement $k_2^{\,2}\to-\mu^2$.

In order to obtain an approximate one-nucleon amplitude we again resort to the zero-range limit of the pion propagators in Eq. (9), i.e., $\mu^2\to\infty$. In that limit the integrations become trivial and the combined result for all the four diagrams in Figs. 3(a)–(d) is

$$T=-i\,\frac{32}{243\pi^3}\left(\frac{k_F}{\mu}\right)^6\frac{ff_\Delta^{\,2}}{m_\Delta-m}\,(\lambda_1+\lambda_2)\vec\sigma\cdot(\vec p_f+\vec p_i)\tau_i\,. \tag{11}$$

This amplitude can be obtained from the interaction (1) with $\lambda=\lambda_{III}$:

$$\lambda_{III}=\frac{32\pi^2}{27\mu^6}\left(\frac{f_\Delta^{\,2}}{4\pi}\right)\frac{\mu}{m_\Delta-m}\,(\lambda_1+\lambda_2)\rho^2$$
$$=\frac{128}{243\pi^2}\left(\frac{f_\Delta^{\,2}}{4\pi}\right)\frac{\mu}{m_\Delta-m}\left(\frac{k_F}{\mu}\right)^6(\lambda_1+\lambda_2)\,. \tag{12}$$

In nuclear matter $\lambda_{III}\simeq0.024$, which is roughly 30% of the corresponding value for λ_{II}.

We finally consider the diagrams in Figs. 3(e) and (f). These can actually be summed to all orders in the internal isobar-hole propagators and then simply represent the self-energy corrections to the pion propagator in the two-body amplitude (4) corresponding to the diagram in Fig. 1(b). They may thus be taken into account by the replacement

$$\frac{1}{k^2+\mu^2}\to\frac{1}{k^2+\mu^2+\Pi_\Delta} \tag{13}$$

in Eq. (4), with Π_Δ being the isobar-hole contribution to the pion self energy[10]:

$$\Pi_\Delta=-\frac{f_\Delta^{\,2}}{4\pi}\left(\frac{32\pi}{9}\right)\frac{\rho k^2}{\mu^2(m_\Delta-m)}\,. \tag{14}$$

This self-energy expression is reduced by taking into account the isobar-hole interaction in the one-pion exchange approximation to[11]

$$\Pi_\Delta\to\Pi_\Delta\left(1-\frac12\,\frac{\Pi_\Delta}{k^2}\right)^{-1}, \tag{15}$$

if the nuclear pair correlation function is taken into account to the minimal extent that it eliminates the δ-function terms in the interaction. The main role of this self energy will be to improve the accuracy of the zero-range approximation used in the other many-body diagrams above. Since the magnitude of Π_Δ in (14) and (15) is $\Pi_\Delta \sim -0.6k^2$, it reduces the kinetic energy terms in the pion propagators to less than half of its unmodified value. We therefore feel that the zero-energy approximation should be accurate anough for a first estimate of the medium corrections to the S-wave ΠNN vertex in nuclei.

The net three-body correction to the parameter λ is thus that given in Eq. (12), and it is again necessarily an overestimate as hadronic form factors and nuclear wave function correlations will reduce it. With the same argument as used above to estimate the form factor correction to λ_{II}, we would find λ_{III} reduced by at least ~10% due to the form factors.

The relative smallness of λ_{III} compared to λ_{II} in nuclear matter suggests that higher order rescattering mechanisms will be of little significance. In any case the large value of λ_{II} compared to the "free nucleon" value $\lambda = \mu/2m$ shows that the effective one-body S-wave pion absorption operator (1) will depend strongly on the nuclear density. The interaction strength will thus vary from nucleus to nucleus, and there will be no possibility of determining a universal value for λ by pionic stripping and knockout reactions. It appears that the only realistic approach to the S-wave pion-nucleus interaction will be to take into account its two-nucleon and many-body components explicitly.

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